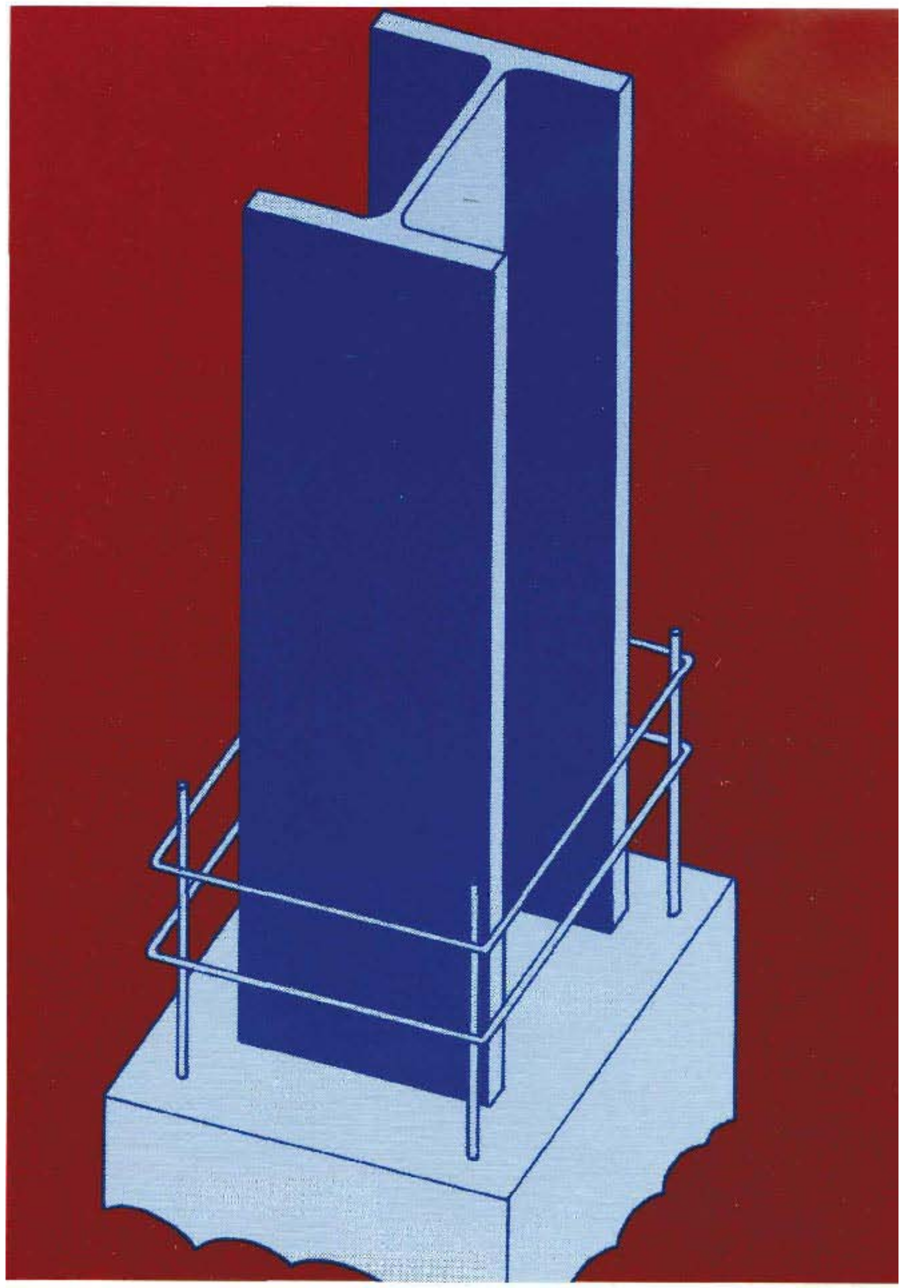
Steel Design Guide Series

Load and Resistance Factor Design of W-Shapes *Encased in Concrete*



Steel Design Guide Series

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Load and Resistance Factor Design of W-Shapes Encased in Concrete (None -Table Version)

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AMERICAN INSTITUTE OF STEEL CONSTRUCTION

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PREFACE

This booklet was prepared under the direction of the Committee on Research of the American Institute of Steel Construction, Inc. as part of a series of publications on special topics related to fabricated structural steel. Its purpose is to serve as a supplemental reference to the AISC Manual of Steel Construction to assist practicing engineers engaged in building design.

The design guidelines suggested by the authors that are outside the scope of the AISC Specifications or Code do not represent an official position of the Institute and are not intended to exclude other design methods and procedures. It is recognized that the design of structures is within the scope of expertise of a competent licensed structural engineer, architect, or other licensed professional for the application of principles to a particular structure.

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LOAD AND RESISTANCE FACTOR DESIGN OF W-SHAPES ENCASED IN CONCRETE

INTRODUCTION

Structural members comprised of steel shapes in combination with plain or reinforced concrete have been utilized by engineers for many years. Early structures simply took advantage of the protection that the concrete afforded to the steel shapes for resistance to fire and corrosion. But research on the strength of such members was conducted in the early 1900s,[1] and design provisions were formulated by 1924.[2] More recently, with the advent of modern composite frame construction in high rise buildings, engineers developed new rational methods to take advantage of the stiffening and strengthening effects of concrete and reinforcing bars on the capacity of encased steel shapes.

This Guide presents design tables for composite columns, developed under the sponsorship of the American Institute of Steel Construction (AISC) as an aid to the practicing structural engineer in the application of the AISC Load and Resistance Factor Design (LRFD) Specification for Structural Steel Buildings.[3] The information presented supplements that found in the AISC LRFD Manual.[4] Background on the LRFD criteria for composite columns may be found in References 5 and 6. Engineers interested in Allowable Stress Design (ASD) are encouraged to consider the procedure developed previously by the Structural Stability Research Council (SSRC).[7] The SSRC procedure is not presently included in the AISC ASD Specification.[8]

The reader is cautioned that independent professional judgment must be exercised when data or recommendations set forth in this Guide are applied. The publication of the material contained herein is not intended as a representation or warranty on the part of the American Institute of Steel Construction, Inc.—or any person named herein—that this information is suitable for general or particular use, or freedom from infringement of any patent or patents. Anyone making use of this information assumes all liability rising from such use. The design of structures should only be performed by or under the direction of a competent licensed structural engineer, architect, or other licensed professional.

SCOPE

This Guide is specifically for composite columns comprised of rolled wide flange shapes encased in reinforced structural concrete with vertical deformed reinforcing bars and lateral ties. Composite columns are defined in Section I1 of the LRFD Specification as a "steel column fabricated from rolled or built-up steel shapes and encased in reinforced structural concrete or fabricated from steel pipe or tubing and filled with structural concrete." Further, the Specification requires in Section I2.1 that the cross sectional area of the steel shape comprise at least four percent of the total composite cross section. The Commentary to the Specification states that when the steel shape area is less, the column should be designed under the rules for conventional reinforced concrete columns.

Part 1 of this Guide includes a discussion of composite frame construction, practical uses of composite columns, their advantages and limitations, and a review of important practical design considerations. A summary of the pertinent LRFD rules is presented and compared to other methods. A set of suggested design details is given in Part 2, showing placement of reinforcing bars and ties, as well as treatment of joints and base plates. Five design examples are given in Part 3 to illustrate how the tables were derived and how they are applied. Finally, a comprehensive set of tables is presented in Part 4 to assist the designer in the rapid selection of the most economical section to resist required values of factored load and moment.

PART 1: USE AND DESIGN OF COMPOSITE COLUMNS

Composite Frame Construction

Although engineers since the 1930s have encased structural steel shapes in concrete for fireproofing and corrosion protection, it was not until the development and popularity of modern composite frame construction in the 1960s that composite columns again became a common and viable structural member type. The late Dr. Fazlur Khan, in his early discussions of structural systems for tall buildings, first proposed the concept of a composite frame system[9, 10] utilizing composite columns as part of the overall wind and earthquake resisting frame. Since that time composite frame construction has been adopted for many high rise buildings all over the world. Its usage, with the composite column as the key element, is well documented in the work of the Council on Tall Buildings and numerous other publications.[11-15]

The term "composite frame structure" describes a building employing concrete encased steel columns and a composite floor system (structural steel and concrete filled steel deck).

The bare steel columns resist the initial gravity, construction, and lateral loads until such time as the concrete is cast around them to form composite columns capable of resisting the total gravity and lateral loads of the completed structure. In a composite frame building, the structural steel and reinforced concrete combine to produce a structure having the advantages of each material. Composite frames have the advantage of speed of construction by allowing a vertical spread of the construction activity so that numerous trades can engage simultaneously in the construction of the building. Inherent stiffness is obtained with the reinforced concrete to more easily control the building drift under lateral loads and reduce perception to motion. The light weight and strength obtained with structural steel equates to savings in foundation costs.

Traditionally in steel framed buildings or reinforced concrete buildings, stability and resistance to lateral loads are automatically provided as the structure is built. Welded or bolted moment connections are made or braces are connected between columns in a steel building immediately behind the erection of the steel frame to provide stability and resistance to lateral loads. Shear walls, or the monolithic casting of beams and columns, provide stability and resistance to lateral loads soon after the concrete has cured for reinforced concrete buildings. However, for composite frame structures, the final stability and resistance to design lateral loads is not achieved typically until concrete around the erection steel frame has cured, which typically occurs anywhere from a minimum of six to as much as 18 floors behind the erection of the bare steel frame. This sequence of construction is shown-schematically in Fig. 1. Thus, as discussed subsequently, *temporary lateral bracing of the uncured portion of the frame will typically be required.*

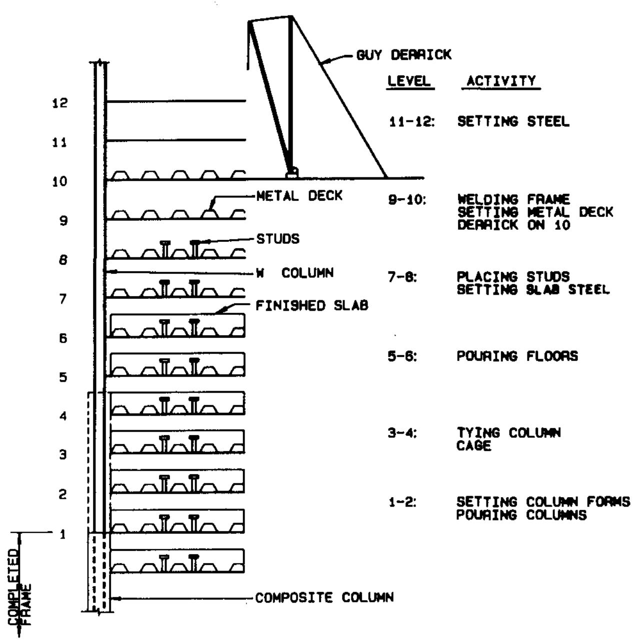

Fig. 1. Composite-frame construction sequence.

Practical Uses of Composite Columns

Practical applications for the use of composite columns can be found in both low rise and high rise structures. In low rise structures such as a covered playground area, a warehouse, a transit terminal building, a canopy, or porte cochere, it may be necessary or desirable to encase a steel column with concrete for aesthetic or practical reasons. For example, architectural appearance, resistance to corrosion, or protection against vehicular impact may be important. In such structures, it may be structurally advantageous to take advantage of the concrete encasement of the rolled steel shape that supports the steel roof structure by designing the member as a composite column resisting both gravity and lateral loads.

In high rise structures, composite columns are frequently used in the perimeter of "tube" buildings where the closely spaced columns work in conjunction with the spandrel beams (either steel or concrete) to resist the lateral loads. In some recent high rise buildings, giant composite columns placed at or near the corners of the building have been utilized as part of the lateral frame to maximize the resisting moment provided by the building's dead load. Composite shear walls with encased steel columns to carry the floor loads have also been utilized in the central core of high rise buildings. Frequently, in high rise structures where floor space is a valuable and income producing commodity, the large area taken up by a concrete column can be reduced by the use of a heavy encased rolled shape to help resist the extreme loads encountered in tall building design. Sometimes, particularly at the bottom floors of a high rise structure where large open lobbies or atriums are planned, a heavy encased rolled shape as part of a composite column is a necessity because of the large load and unbraced length. A heavy rolled shape in a composite column is often utilized where the column size is restricted architecturally and where reinforcing steel percentages would otherwise exceed the maximum code allowed values.

Advantages, Disadvantages, and Limitations

Some of the advantages of composite columns are as follows:

1. Smaller cross section than required for a conventional reinforced concrete column.
2. Larger load carrying capacity.
3. Ductility and toughness available for use in earthquake zones.
4. Speed of construction when used as part of a composite frame.
5. Fire resistance when compared to plain steel columns.
6. Higher rigidity when part of a lateral load carrying system.
7. Higher damping characteristics for motion perception in tall buildings when part of a lateral load carrying system.

8. Stiffening effect for resistance against buckling of the rolled shape.

There are also, of course, some disadvantages and limitations. In high rise composite frame construction, design engineers sometimes have difficulty in controlling the rate and magnitude of column shortening of the composite column with respect to adjacent steel columns or shear walls. These problems are exacerbated by the wide variation in construction staging often experienced in the zone between the point where the steel erection columns are first erected and the point where concrete is placed around the steel to form the composite column. This variation in the number of floors between construction activities has made it difficult to calculate with accuracy the effect of column shortening. Creep effects on the composite columns with respect to the all-steel core columns, or between shears walls, can also be troublesome to predict for the designer. The net effect of these problems can be floors that are not level from one point to another. One solution to these problems has been the measurement of column splice elevations during the course of construction, with subsequent corrections in elevation using steel shims to compensate for differences between the calculated and measured elevation.

As with any column of concrete and reinforcing steel, the designer must be keenly aware of the potential problems in reinforcing steel placement and congestion as it affects the constructability of the column. This is particularly true at beam-column joints where potential interference between a steel spandrel beam, a perpendicular floor beam, vertical bars, joint ties, and shear connectors can all cause difficulty in reinforcing bar placement and lead to honeycombing of the concrete. Careful attention must be given to the detailing of composite columns by the designer. Analytical and experimental research is needed in several aspects of composite column design. One area requiring study is the need, or lack thereof, of a mechanical bond between the steel shape and the surrounding concrete. Several papers[16,17] have discussed this question, but additional work is required to quantify the need for shear connectors with a practical design model for routine design office use. There presently is a question about transfer of shear and moment through a beam-column joint. This concern is of particular importance for seismic regions where large cyclical strain reversals can cause a serious degradation of the joint. Initial research has been completed at the University of Texas at Austin[24] and is ongoing at Cornell University on physical test models to study various joint details in composite columns.

Practical Design Considerations

Fire Resistance

Composite columns, like reinforced concrete columns, have an inherent resistance to the elevated temperatures produced in a fire by virtue of the normal concrete cover to the reinforcing steel and structural steel. It is standard practice to provide a minimum of one and one-half inch of concrete cover to the reinforcing steel of a composite column (concrete cover is specified in ACI 318-89 Section 7.7.1).[18] Chapter 43 of the Uniform Building Code states that reinforced concrete columns utilizing Grade A concrete (concrete made with aggregates such as limestone, calcareous gravel, expanded clay, shale, or others containing 40 percent or less quartz, chert, or flint) possess a four-hour rating with one and one-half inch cover. A four-hour rating is the maximum required for building structures.

Tables of fire resistance rating for various insulating materials and constructions applied to structural elements are published in various AISI booklets[19,20,21] and in publications of the Underwriters Laboratory, Inc.

Longitudinal Reinforcing Bar Arrangement

Composite columns can take on just about any shape for which a form can be made and stripped. They can be square, rectangular, round, triangular, or any other configuration, with just about any corresponding reinforcing bar arrangement common to concrete columns. For use in composite frame construction, however, square or rectangular columns

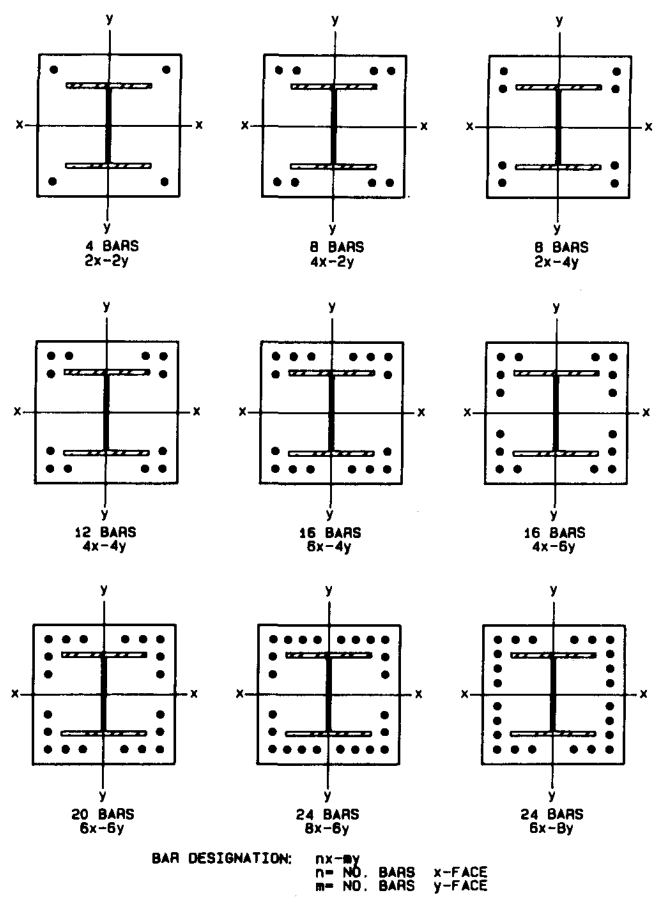

Fig. 2. Longitudinal bar arrangement in composite columns.

are the most practical shape, with bar arrangements tending to place the vertical reinforcing bars at or near the four corners of the column. Figure 2 shows preferred arrangements which allow spandrel beams and a perpendicular floor beam to frame into the encased steel shape without interrupting the continuous vertical bars. Such arrangements also generate the maximum design capacity for the column.

Although there are no explicit requirements for longitudinal bar spacing in the LRFD Specification, it is advisable to establish minimum limits so that concrete can flow readily in spaces between each bar and between bars and the encased steel shape.

Minimum spacing criteria will also prevent honeycombing and cracks caused by high bond stresses between bars. Past experience with reinforced concrete columns has shown that the requirements established by the ACI 318 Code have provided satisfactory performance. These spacing and cover requirements have been used in the formulation of this design aid and as diagramed in Fig. 3 and listed below:

1. Minimum concrete cover over vertical bars and ties shall be 1½-in. (LRFD Specification, Section I2.1.b).
2. Clear distance between longitudinal bars shall not be less than 1½ bar diameters or 1½-in. minimum (ACI 318-89 Section 7.6.3).

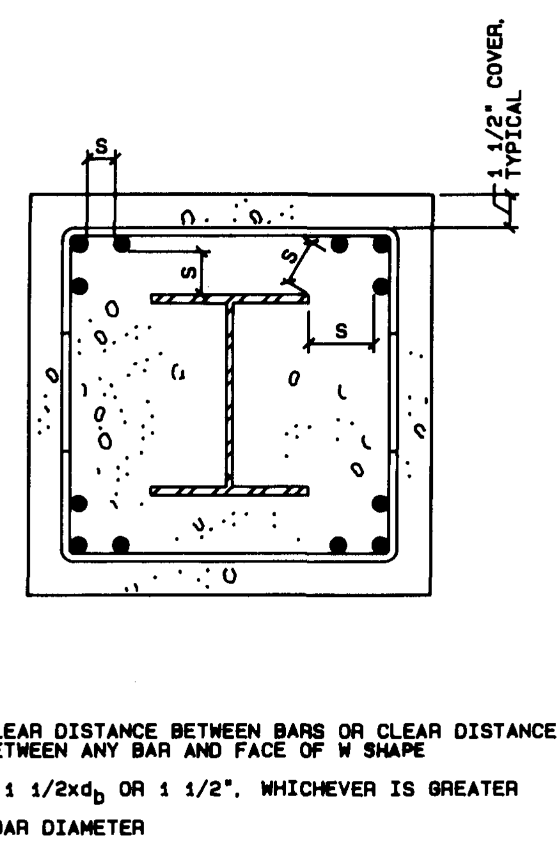

Fig. 3. Composite column cover and bar spacing requirements.

3. The clear distance limitations apply also to contact lap splices and adjacent bars (ACI 318-89 Section 7.6.4).
4. Clear distance between longitudinal bars and steel shape shall be 1½ bar diameters or 1½-in. minimum.

Ties

Reinforcing steel cages (longitudinal bars and ties) must usually be set after and around the steel column. Because the steel column is erected in an earlier erection sequence, only open U-shaped ties are suitable for composite columns. Ties are used to provide lateral stability of the longitudinal bars and confinement of the concrete. The requirements of the LRFD specification and certain requirements of the ACI 318-89 code not specifically addressed by the LRFD specification should be satisfied as follows:

1. The cross sectional area of the tie shall be at least 0.007 square inches per inch of tie spacing (LRFD Specification I2.1.b).
2. The spacing of the ties shall not be greater than two-thirds of the least dimension of the cross section (LRFD Specification I2.1.b).
3. The spacing of ties shall not be greater than 16 longitudinal bar diameters or 48 tie bar diameters (ACI 318-89 Section 7.10.5.1).
4. Ties shall be at least #4 in size for #11, #14, #18, and bundled longitudinal bars, and #3 in size for all other bars (ACI 318-89 Section 7.10.5.1).
5. Ties shall be arranged such that every corner and alternate bar shall have lateral support provided by a corner of a tie, with an inclusive angle of not more than 135° and no bar shall be further than 6 inches clear on each side along the tie from such a laterally supported bar (ACI 318-89 Section 7.10.5.3).
6. A lap splice of two pieces of an open tie shall be at least equal to 1.3 times the tensile development length for the specified yield strength (ACI 318-89 Section 12.13.5).

Suggested details for composite column ties are shown in Typical Details 1, 2, and 3 of Part 2.

Longitudinal Reinforcing Bar Splices

The requirements for splicing vertical longitudinal reinforcing bars for composite columns shall follow the same rules as apply for conventional reinforced concrete columns as specified in Chapter 12 of the ACI 318-89 Code. Several additional comments should be made for composite columns. First, additional vertical longitudinal restraining bars (LRFD Specification I2.1.b) should be used between the corners where the continuous load carrying bars are located in composite frame construction. These bars usually cannot be continuous because of interruption with intersecting framing members at the floor line. They are often required to satisfy the spacing requirements for vertical longitudinal bars shown as follows:

The cross section area of longitudinal reinforcement shall be at least equal to 0.007 square inches per inch of bar spacing (LRFD Specification I2.1.b).

Second, it is suggested that, in high rise composite frame construction, the vertical bar splices be located at the middle clear height of the composite column. This point is usually near the inflection point (zero moment) of the column where the more economical compression lap splices or compression butt splices may be used. The more expensive tension lap or tension butt splices may be required if splices are made at the floor line.

A suggested composite column splice detail is shown in Typical Detail 1 of Part 2.

Connection of Steel Beam to Encased Wide Flange

In composite frame construction, steel spandrel beams and/ or perpendicular floor beams often frame into the composite column at the floor level. Sometimes these beams will be simply supported floor beams where conventional double-angle framed beam connections (LRFD Manual, Part 5) or single-plate shear connections may be utilized. More often, however, the steel spandrel beams will be part of the lateral load resisting system of the building and require a moment connection to the composite column. Practicality will often dictate that the larger spandrel beam (frequently a W36 in tall buildings) be continuous through the joint with the smaller erection column (often a small W14) interrupted and penetration welded to the flanges of the spandrel beam. To increase the speed of erection and minimize field welding, the spandrel beam and erection column are often prefabricated in the shop to form "tree columns" or "tree beams" with field connections at the mid-height of column and midspan of spandrel beam using high strength bolts. See Typical Detail 5, Part 2.

The engineer must concern himself with the transfer of forces from the floor beams to the composite column. For simply supported beams not part of the lateral frame, the simplest method to transfer the beam reaction to the composite column is through a standard double-angle or single-plate shear connection to the erection column. It is then necessary to provide a positive shear connection from the erection column to the concrete along the column length to ensure transfer of the beam reaction to the composite column cross section. The simplest method to accomplish this is by the use of standard headed shear connectors, preferably shop welded to the wide flange column. For moment connected spandrel beams, the beam shear and unbalanced moment must be transferred to the composite column cross section. Different transfer mechanisms have been tested at the University of Texas at Austin.[24]

Several suggested details are shown in Details 1 and 2 of Part 2.

Shear Connectors

As discussed in the previous section, it is necessary to provide a positive shear connection transfer from the floor beam to the encased steel column when the beam connection is made directly to the encased steel column. It is likely that a significant portion of this reaction can be transferred in bond between the encased section and the concrete as reported in Reference 14. An estimate of this value can be made from Equation 5 of Reference 16 which is based on the results of a limited number of push tests in which a steel column is encased in a concrete column.

$$P_{sl} = \frac{3.6 b_f (0.09 f_c' - 95)\, l_e}{k}$$

where

P_{sl} = allowable load for the encased shape, lb
b_f = steel flange width of encased shape, in.
f_c' = concrete compressive strength, psi
l_e = encased length of steel shape, in.
k = constant $\approx$ 5

Converting to an average ultimate bond stress "u," using only the flange surfaces as being effective and applying a safety factor of five as reported in the tests.

$$u = \frac{P_{sl} \times 5}{4 b_f l_e} = \frac{3.6 \times b_f (.09 f_c' - 95)\, l_e}{5} \times \frac{5}{4 b_f l_e}$$

$$u = 0.9\,(0.9 f_c' - 95), \text{ average ultimate bond stress, psi}$$

Consider a typical case of a W14x90 encased column in 5,000 psi concrete with a floor-to-floor height (h_o) of 13 feet. The average ultimate bond stress is

$$u = 0.9\,(0.09 \times 5{,}000 - 95) = 320 \text{ psi}$$

The ultimate shear force that could be transferred by bond is

$$= u \times h_o \times 4 b_f = \frac{320 \times (13 \times 12) \times (4 \times 14.5)}{1{,}000} = 2{,}895 \text{ kips}$$

These results indicate that typical floor reactions on the composite column could be easily transferred by bond alone.

The above discussion considered the case where axial load alone is transferred from the encased steel section to the concrete. For beam-columns where high bending moments may exist on the composite column, the need for shear connectors must also be evaluated. Until such time as research data is provided, the following simplistic evaluation may be made. Assume a situation where a composite column is part of a lateral load resisting frame with a point of inflection at mid-column height and a plastic neutral axis completely outside the steel cross section (similar to Fig. 4 except for plastic neutral axis location). An analogy can be made between this case and that of a composite beam where shear connectors are provided uniformly across the member length between the point of zero moment and maximum moment. The ultimate axial force to be transferred between the encased steel column and the concrete over the full column height is $2AF_y$ where A is the steel column area and F_y is its yield strength. Assuming a bond strength is available in this case similar to the case of the push test discussed above, then shear connectors would theoretically be required when $2AF_y$ is greater than the ultimate bond force. In the previous example, assume an A36 W14×90 erection column is used. Then,

$$2AF_y = 2 \times 26.5 \times 36 = 1{,}908 \text{ kips}$$

This is less than the available shear transfer from bond, which was calculated as 2,895 kips

Again, it is shown that bond stress alone can transfer the shear between the encased shape and the concrete, assuming no loss in bond occurs as a result of tensile cracking at high moments.

The composite beam-column design tables presented in Part A assume a nominal flexural strength based on the plastic stress distribution of the full composite cross section. To validate this assumption, the LRFD specification commentary in Section 14, requires a transfer of shear from the steel to the concrete with shear connectors. Therefore, until further research is conducted on the loss of bond between the encased steel section and the concrete, and until more comprehensive push tests are run, the following suggestions are made with regard to shear connectors on composite columns:

1. Provide shear connectors on the outside flanges where space permits. Where space does not permit, provide shear connectors on the inside flange staggered either side of the web.
2. Provide shear connectors in sufficient quantity, spaced uniformly along the encased column length and around the column cross section between floors, to carry the greater of the following minimum shear transfer forces as applicable:
   a. The sum of all beam reactions at the floor level.
   b. Whenever the ratio of the required axial strength to the factored nominal axial strength, $P_u / \phi_c P_n$, is less than 0.3, a force equal to F_y times the area of steel on the tensile side of the plastic neutral axis in order to sustain a moment equal to the nominal flexural strength of the composite cross section. The ratio 0.3 is used as an arbitrary value to distinguish a composite column subjected to predominantly axial load from one subjected to predominately moment. Consideration must be given to the fact that this moment is reversible.
3. The maximum spacing of shear connectors on each flange is suggested to be 32 inches.

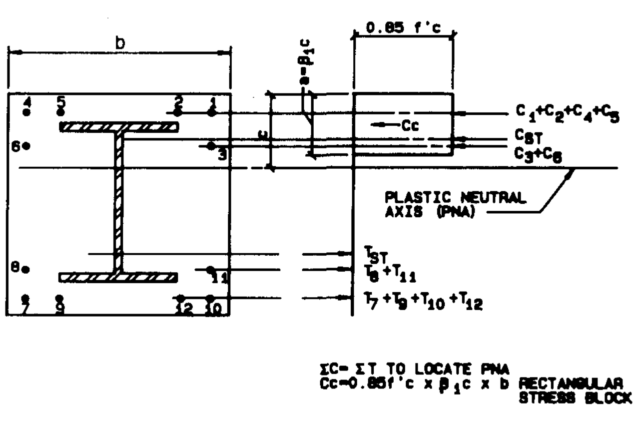

Fig. 4. Plastic stress distribution in composite columns.

If minimum shear connectors are provided according to the guidelines identified herein, it is reasonable to assume compatibility of strains between concrete and encased steel to permit higher strains than 0.0018 under axial load alone. This strain level has been identified in Reference 7 and LRFD Commentary, Section 12.1, as a point where unconfined concrete remains unspalled and stable. Therefore, a slight increase in the maximum usable value of reinforcing steel stress from 55 ksi, corresponding to 0.0018 axial strain, to 60 ksi, the yield point of ASTM A615 Grade 60 reinforcing steel, would seem to be justified. Such an approach has been adopted in this Guide. The use of shear connectors also allows the full plastic moment capacity to be counted upon when $P_u / (\phi_c M_n)$ is less than 0.3 (LRFD Commentary, I4) instead of the reduction specified in LRFD Specification, Section I4.

Suggested details for shear connectors on composite columns are shown in Typical Details 1 and 2 of Part 2.

Base Plate

Normally a base plate for the encased steel column of a composite column is specified to be the minimum dimension possible to accommodate the anchor bolts anchoring it to the foundation during the erection phase. In doing so, the base plate will interfere the least possible amount with dowels coming up from the foundation to splice with the longitudinal vertical bars of the composite column. The design engineer must provide dowels from the composite column to the foundation to transmit the column load in excess of the allowable bearing stress on the foundation concrete ($\phi_c \times 0.85 \times f_c'$) times the effective bearing area (the total composite column area less the area of the encased wide flange column base plate). In some cases, depending on the base plate size, it may be necessary to add additional foundation dowels to adequately transmit the load carried by the concrete of the composite column. A typical base plate detail is shown in Typical Detail 4, Part 2. A composite column base plate example is included as Example 5, Part 3.

Erection and Temporary Wind Bracing During Composite Frame Construction

Historically, a structural steel erector is accustomed to working with a steel framed structure that is stabilized as the frame is constructed with moment connections or permanent cross bracing. Composite frames many times are not stable and not fully able to carry lateral loads until after the concrete is poured and cured many floors behind. Because of this fact, it is incumbent on the engineer-of-record to state the assumptions of bare steel frame stability in the contract documents. Either he designs and details the necessary temporary bracing on the drawings or requires the erector to engage a structural engineer to provide it. The engineer-of-record is the most appropriate person to provide this service by virtue of his knowledge of the loads and familiarity with the overall structure. Additional discussions about the design responsibility of steel frames during erection may be found in the AISC Code of Standard Practice.[22] A discussion of composite frames during erection may be found in Reference 15.

Load and Resistance Factor Design (LRFD) of Composite Columns

To qualify as a composite column under the LRFD Specification design procedure, the following limitations must be satisfied as defined in Section 12.1:

1. The cross sectional area of the steel shape, pipe, or tubing must comprise at least four percent of the total composite cross section.
2. Concrete encasement of a steel core shall be reinforced with longitudinal load carrying bars, longitudinal bars to restrain concrete, and lateral ties. Longitudinal load carrying bars shall be continuous at framed levels; longitudinal restraining bars may be interrupted at framed levels. The spacing of ties shall be not greater than two-thirds of the least dimension of the composite cross section. The cross sectional area of the transverse and longitudinal reinforcement shall be at least 0.007 in.^{2} per inch of bar spacing. The encasement shall provide at least 1½-in. of clear cover outside of both transverse and longitudinal reinforcement.
3. Concrete shall have a specified compressive strength f_c' of not less than 3 ksi nor more than 8 ksi for normal weight concrete, and not less than 4 ksi for lightweight concrete.
4. The specified minimum yield stress of structural steel and reinforcing bars used in calculating the strength of a composite column shall not exceed 55 ksi.

The required design strength P_u of axially loaded composite columns is defined in the LRFD Specification, Section E2, with modification of certain terms according to Section I2.2. These rules are summarized as follows:

$P_u = \phi_c P_n$, required axial strength

$$P_n = A_s F_{cr}, \text{ nominal axial strength} \qquad \text{(E2-1 modified)}$$

For $\lambda_c \le 1.5$,

$$F_{cr} = (0.658^{\lambda_c^2}) F_{my} \qquad \text{(E2-2 modified)}$$

For $\lambda_c > 1.5$,

$$F_{cr} = \frac{0.877}{\lambda_c^2} F_{my} \qquad \text{(E2-3 modified)}$$

$$\lambda_c = \frac{Kl}{r_m \pi} (F_{my} / E_m)^{1/2} \qquad \text{(E2-4 modified)}$$

ϕ_c = resistance factor for compression = 0.85
A_s = gross area of steel shape
F_{my} = modified yield stress
$= F_y + c_1 F_{yr} (A_r / A_s) + c_2 f_c' (A_c / A_s)$, ksi (I2-1)
E_m = modified modulus of elasticity
$= E + c_3 E_c (A_c / A_s)$, ksi (I2-2)
F_y = specified yield stress of structural steel column, ksi
E = modulus of elasticity of steel, ksi
K = effective length factor
l = unbraced length of column, in.
r_m = radius of gyration of steel shape in plane of buckling, except that it shall not be less than 0.3 times the overall thickness of the composite cross section in the plane of buckling, in.
A_c = net concrete area $= A_g - A_s - A_r$, in.^{2}
A_g = gross area of composite section, in.^{2}
A_r = area of longitudinal reinforcing bars, in.^{2}
E_c = modulus of elasticity of concrete $= w_c^{1.5} (f_c')^{1/2}$, ksi
w_c = unit weight of concrete, lbs./ft^3
f_c' = specified compressive strength of concrete, ksi
F_{yr} = specified minimum yield stress of longitudinal reinforcing bars, ksi
c_1 = 0.7
c_2 = 0.6
c_3 = 0.2

The interaction of axial compression and flexure in the plane of symmetry on composite members is defined in Section H1.1, H1.2, and I4 as follows:

For $P_u / \phi_c P_n \ge 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0 \qquad \text{(H1-1a)}$$

For $P_u / \phi_c P_n < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0 \qquad \text{(H1-1b)}$$

P_u = required compressive strength, kips
P_n = nominal compressive strength, kips
M_u = required flexural strength, kip-in.
M_n = nominal flexural strength determined from plastic

stress distribution on the composite cross section, kip-in.

ϕ_c = resistance factor for compression = 0.85

ϕ_b = resistance factor for flexure = 0.90

The following information on the determination of the required flexural strength, M_u, is quoted from Section H1.2 of the LRFD Specification, with minor changes in symbols as prescribed in Section I2.

"In structures designed on the basis of elastic analysis, M_u may be determined from a second order elastic analysis using factored loads. In structures designed on the basis of plastic analysis, M_u shall be determined from a plastic analysis that satisfies the requirements of Sects. C1 and C2. In structures designed on the basis of elastic first order analysis the following procedure for the determination of M_u may be used in lieu of a second order analysis:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{H1-2}$$

where

M_{nt} = required flexural strength in member assuming there is no lateral translation of the frame, kip-in.

M_{lt} = required flexural strength in member as a result of lateral translation of the frame only, kip-in.

$$B_1 = \frac{C_m}{(1 - P_u / P_e)} \geq \tag{H1-3}$$

$P_e = A_s F_{my} / \lambda_c^2$ where λ_c is defined by Formula E2-4 with $K \leq 1.0$ in the plane of bending.

C_m = a coefficient whose value shall be taken as follows:

i. For restrained compression members in frames braced against joint translation and not subject to transverse loading between their supports in the plane of bending,

$$C_m = 0.6 - 0.4(M_1 / M_2) \tag{H1-4}$$

where M_1 / M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1 / M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

ii. For compression members in frames braced against joint translation in the plane of loading and subjected to transverse loading between their supports, the value of C_m can be determined by rational analysis. In lieu of such analysis, the following values may be used:

for members whose ends are restrained, C_m = 0.85

for members whose ends are unrestrained, C_m = 1.0

$$B_2 = \frac{1}{1 - \Sigma P_u \left(\frac{\Delta_{oh}}{\Sigma HL} \right)} \tag{H1-5}$$

or

$$B_2 = \frac{1}{1 - \frac{\Sigma P_u}{\Sigma P_e}} \tag{H1-6}$$

ΣP_u = required axial load strength of all columns in a story, kips

Δ_{oh} = translation deflection of the story under consideration, in.

ΣH = sum of all story horizontal forces producing Δ_{oh}, kips

L = story height, in.

$P_e = A_s F_{my} / \lambda_c^2$, kips, where λ_c is the slenderness parameter defined by Formula E2-4, in which the effective length factor K in the plane of bending shall be determined in accordance with Sect. C2.2, but shall not be less than unity."

The nominal flexural strength M_n is determined for the plastic stress distribution on the composite cross section as shown in Fig. 4. The plastic neutral axis is first determined such that there is equilibrium of axial forces in the concrete, reinforcing steel and embedded steel column. The nominal flexural strength M_n is determined as the summation of the first moment of axial forces about the neutral axis. See Example 2, Part 3.

In the determination of the concrete compressive axial force, a concrete compressive stress of $0.85 f_c'$ is assumed uniformly distributed over an equivalent stress block bounded by the edges of the cross section and a straight line parallel to the plastic neutral axis at a distance $a = \beta_1 c$, where c is the distance from the edge of the cross section to the plastic neutral axis, and,

$$\beta_1 = 0.85 \text{ for } f_c' \leq 4 \text{ ksi}$$

$$\beta_1 = 0.85 - 0.05(f_c' - 4) \geq 0.65 \text{ for } f_c' > 4 \text{ ksi}$$

These assumptions are contained in the ACI 318-89 Code (Section 10.2.7.3).

Comparison Between LRFD and Strain Compatibility Methods

Guidelines for the design of composite columns were first introduced into the ACI Building Code in 1971 (ACI 318-71). With the widespread use and popularity of composite columns in the 1970s and 1980s, many engineers designed composite columns according to these principles, which are essentially the same ones used for conventional reinforced concrete columns.

The current rules for designing composite columns by the

ACI approach are found in ACI 318-89, Chapter 10. The method essentially is one based on the assumption of a linear strain diagram across the composite cross section with the maximum failure strain at ultimate load defined as 0.003. With these assumptions, it is possible to generate strength capacities of the cross section for successive assumed locations of the neutral axis. Strains at each location of the cross section are converted to stress for the usual assumption of a linear stress-strain curve for reinforcing steel and structural steel. The first moment of forces in each element of concrete, structural steel, and reinforcing steel is taken about the neutral axis to generate a point (axial load and moment) on an interaction curve.

A comparison between the strain compatibility approach and the LRFD approach is shown in Figs. 5 through 7. Interaction curves (axial load vs. moment) are plotted covering the wide range of composite column sizes (28×28 in., 36×36 in., 48×48 in.) steel column sizes (minimum of four percent of the composite column cross section to maximum W14×730) and reinforcing steel percentages (one percent to four percent) that are likely to be found in practice. Examination of these figures reveals the following comparison:

1. The ACI approach yields curves that are parabolic in nature while the AISC curves are essentially bilinear.
2. The two methods yield pure moment capacities that are very close to each other. The maximum difference is approximately 15 percent with most values much closer than that. LRFD in all cases predicts higher moment values.
3. The two methods yield pure axial load capacities that are reasonably close when the steel column constitutes a small part of the total column capacity, but are significantly different as the steel column becomes larger. With larger steel column sizes, the LRFD approach yields axial capacities as much as 30 percent larger than ACI. This comparison, however, is not very meaningful because the ACI approach essentially does not recognize pure axially loaded columns with its minimum eccentricity provisions.
4. Large differences in capacity are predicted (as much as 50 percent) for composite columns having small steel columns. The ACI method yields significantly larger axial loads for a given moment than the LRFD method. This difference is most striking in the intermediate range of the curve.
5. With larger steel columns, the LRFD curve is mostly above (predicts higher values) the ACI curve. As the steel column section becomes lighter, the ACI curve tends to be above the LRFD curve, particularly in the middle ranges of eccentricity.
6. It can generally be stated that, as the steel column becomes a larger portion of the total column capacity, design economy can be realized by designing using the LRFD approach. When the steel column becomes

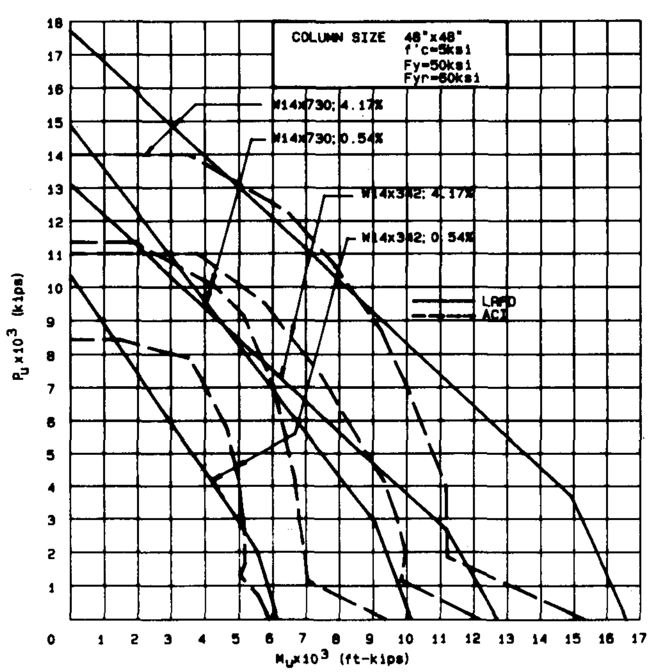

Fig. 5. Interaction curve comparisons ACI vs. LRFD.

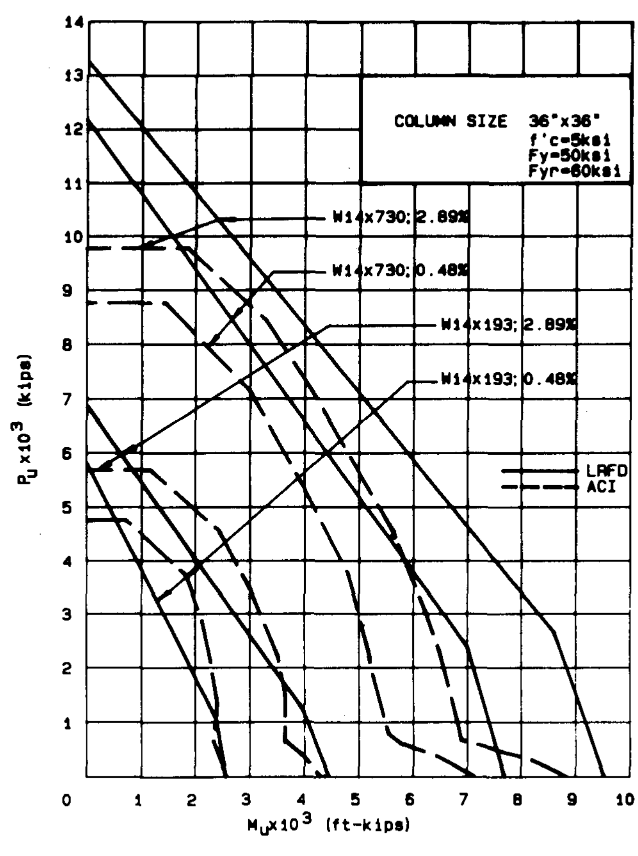

Fig. 6. Interaction curve comparisons ACI vs. LRFD.

smaller (the column is more like a conventional concrete column), the ACI method is more economical in design.

Reference 23 also presents a comparison of design methods.

Description of the Composite Beam-Column Load Tables

Design tables are presented in Part 4 of this Guide to assist the engineer in the rapid selection of the most economical composite column to resist factored values of axial load and moment. The tables are based on the LRFD Specification requirements outlined in the previous sections. The tables have been set up to follow the general format of the LRFD Manual,[4] including the column tables in Part 2 (Axial Loaded Steel Columns) and Part 4 (Axially Loaded Composite Columns) of the Manual, because these are already familiar to most design engineers. The tables indicate the following parameters from which the engineer can select a design (Refer to sample table at beginning of Part 4 of this Guide):

Item 1: Composite Column Size *($b \times h$,* in.). The composite column size *($b \times h$)* is indicated in inches in the upper right comer of the table. Note that the x- x axis is always the strong axis of the steel column and is in the direction of *b.* The y-y axis is always the weak axis of the steel column and is in the direction of *h.* The table covers square and rectangular sizes varying from 16 inches to 36 inches in four-inch increments.

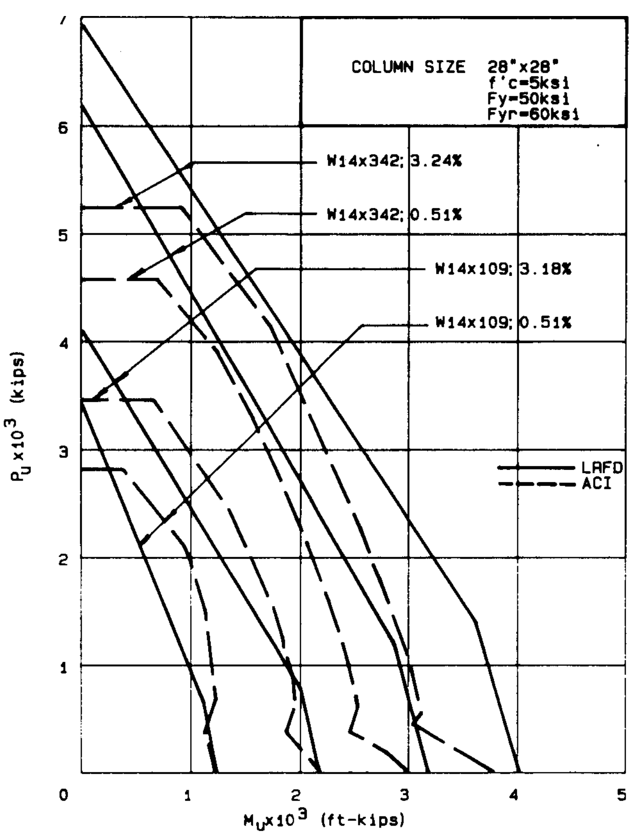

Fig. 7. Interaction curve comparisons ACI vs. LRFD.

Item 2: Concrete Strength (f'_c, ksi). Concrete compression strength (f'_c) is indicated in the top right corner for 3 and 8 ksi. All concrete is assumed to be normal weight concrete weighing 145 pcf. Linear interpolation can be used for concrete strengths between 3 and 8 ksi.

Item 3: Reinforcing Bar Yield Strength *(F_{yr},* ksi). All longitudinal and transverse reinforcing steel in the table is based on ASTM A615 Grade 60 reinforcing steel.

Item 4: Steel Column Size. Steel column size is listed across the top of the table. Sizes tabulated include all W8, W10, W12, and W14 wide flange shapes that are listed in the steel column tables in Part 4 of the LRFD manual. They include W8 (35 to 67), W10 (39 to 112), W12 (50 to 336), and W14 (43 to 426).

Item 5: Steel Grade *(F_y,* ksi). Steel grade is presented across the top of the page for both A36 and Grade 50 steel.

Item 6: Reinforcement. Information on column reinforcement is indicated in the extreme left column and includes the percentage of vertical steel, area of steel *(A_r,* in.^{2}) number, size of bar, pattern of vertical steel, and lateral tie size and spacing (see Fig. 2 for notation). The table covers steel percentages as close as practical to 0.5 percent, 1 percent, 2 percent, 3 percent, and 4 percent steel. If zeroes are tabulated, it indicates steel cover or spacing requirements could not be satisfied for the steel percentage indicated. Bar arrangements and their designations are shown in Fig. 2.

Item 7: Unbraced Length *(KL,* ft). Axial load capacities are tabulated for unbraced lengths of 0, 11, 13, 17, 21, 25, and 40 feet.

Item 8: Axial Design Strength (Nominal Axial Strength times Resistance Factor, $\phi_c P_n$, kips). For each unbraced length, *KL,* equations E2-1, E2-2, E2-3, and E2-4 are used to calculate the nominal axial strength which is multiplied by $\phi_c = 0.85$ and tabulated in the column marked 8.

Item 9, 10, and 11: Available Required Flexural Strength (Uniaxial Moment Capacity, M_{ux}, M_{uy}, ft-kips). For each ratio of applied factored axial load to ϕ_c times the nominal axial capacity, $P_u / \phi P_n$, available uniaxial moment capacity is tabulated by solving equation H1-1a or H1-1b as applicable. *Note that these moment capacities are uniaxial capacities and are applied independently. Biaxial moment capacities are not tabulated.*

Item 12: Euler Buckling Term (C_{ex}, C_{ey}, kip-ft^2). The second order moment, M_u, can be taken directly from a second order elastic analysis, or it can be calculated from a first order elastic analysis by using LRFD equations H1-1 through H1-6. To aid the designer in such a calculation, the terms C_{ex} and C_{ey} are tabulated for each column configuration. The following definitions apply.

$$C_{ex} = \frac{P_{ex}(K_x L_x)^2}{10{,}000} \quad C_{ey} = \frac{P_{ey}(K_y L_y)^2}{10{,}000}$$

Thus, the Euler buckling load needed for the calculation is simply

$$P_e = 10{,}000 C_e / (KL)^2$$

Item 13: Radius of Gyration (r_{mx}, r_{my}, in.). To compare the axial design strength for buckling about each axis, and to assist the designer in determining column capacity for unbraced lengths not shown in the table, values of r_{mx} and r_{my} are tabulated for each column configuration.

Note that the development of the moment capacities listed in the tables is based on a numerical calculation of the contribution of the encased shape, the precise number and location of reinforcing bars as prescribed in the bar arrangements of Fig. 2, and the concrete. This is in lieu of the approximate plastic moment capacity expression prescribed by the LRFD Commentary equation C-I4-1. The approximate expression was used in the moment capacities tabulated in the composite column tables presently in the LRFD Manual and will result in some differences when compared to the more precise method used in the new composite beam-column tables in this Guide.

The following factors should be considered in the use of the tables:

1. Where zeroes exist in the tables, no bar pattern from the configurations considered in Fig. 2 exists that would satisfy bar cover and spacing requirements between bars, or between bars and the surface of the encased steel column (Refer to Fig. 3).
2. Moment capacity tabulated is the uniaxial moment capacity considering each axis separately.
3. Only column configurations conforming to all the limitations in the LRFD Specification (Section I2.1) are tabulated.
4. Capacities shown are only applicable to the bar arrangements shown in Fig. 2.
5. The designer must determine in each case that necessary clearances are available for beams framing into the steel column without interrupting the vertical bars.
6. Linear interpolation can be used to determine table values for concrete strengths between 3 and 8 ksi.

Specific instruction for using the tables are given at the beginning of the tables, Part 4 of this Guide. The background for the development of the tables is presented in Examples 1 and 2, Part 3 of this Guide.

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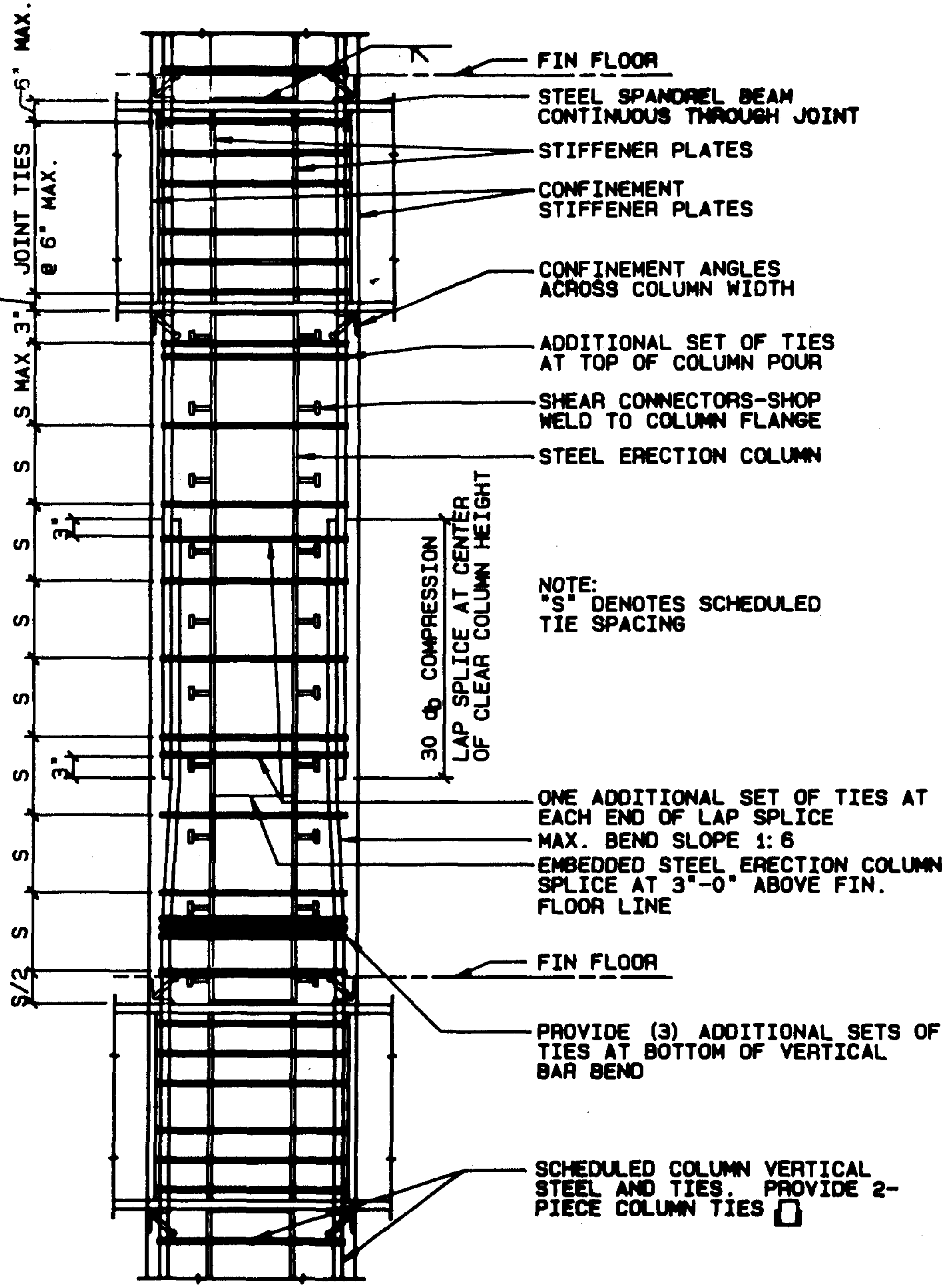
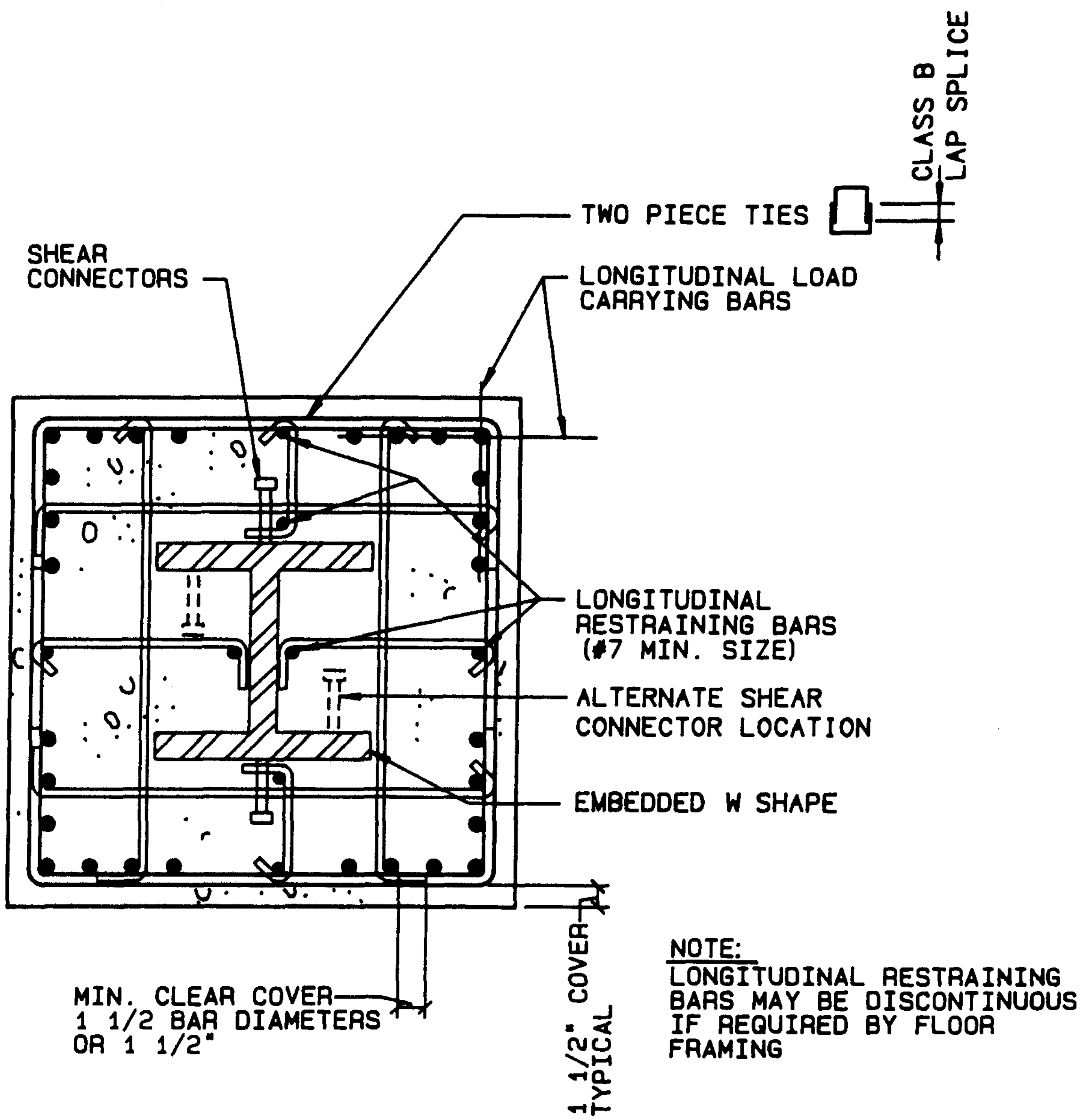
NOMENCLATURE

A_1 = Area of base plate, in.^{2}
A_2 = Full cross sectional area of concrete support, in.^{2}
A_c = Net concrete area, in.^{2}
A_g = Gross area of composite section, in.^{2}
A_H = Area of H-shaped portion of base plate, in.^{2}
A_r = Area of reinforcing bars, in.^{2}
A_s = Gross area of steel shape, in.^{2}
B = Base plate width, in.
B_1 B_2 = Factors used in determining M_u for combined bending and axial forces when first order analysis is employed
C = Compression force in reinforcing bar, kips
C_c = Compressive force in concrete, kips
C_e = Factor for calculating Euler buckling strength, kip-ft^2
C_m = Coefficient applied to bending term in interaction formula
E = Modulus of elasticity of steel (29,000 ksi)
E_c = Modulus of elasticity of concrete, ksi
E_m = Modified modulus of elasticity, ksi
F_{cr} = Critical stress, ksi
F_{my} = Modified yield stress, ksi
F_y = Specified minimum yield stress of the type of steel being used, ksi
F_{yr} = Specified minimum yield stress of reinforcing bars, ksi
H = Horizontal force, kips
K = Effective length factor for prismatic member
L = Unbraced length of member measured between the center of gravity of the bracing members, in.
L = Story height, in.
M_1 = Smaller moment at end of unbraced length of beam column, kip-in.
M_2 = Larger moment at end of unbraced length of beam column, kip-in.
M_{lt} = Required flexural strength in member due to lateral frame translation, kip-in.
M_n = Nominal flexural strength, kip-in.
M_{nt} = Required flexural strength in member assuming there is no lateral translation of the frame, kip-in.
M_u = Required flexural strength, kip-in.
N = Base plate length, in.
P_e = Euler buckling strength, kips
P_n = Nominal axial strength, kips
P_o = Factored load contributory to area enclosed by steel shape, kips
P_s = Factored axial load resisted by steel shape, kips
P_{sl} = Service load for encased shape limited by bond stress, lbs
P_u = Required axial strength, kips
R_u = Ratio of required axial strength to factored nominal axial strength
T = Tension force in reinforcing bar, kips
T_{ST} = Tension force in steel shape, kips
a = Depth of compression block of concrete in composite column, in.
b = Overall width of composite column, in.
b_f = Flange width, in.
c = Distance to outer fiber from plastic neutral axis, in.
c_1, c_2, c_3 = Numerical coefficients for calculating modified properties
d = Overall depth of member, in.
f_c' = Concrete compressive stress, psi or ksi, as applicable
h = Overall depth of composite column, in.
h_o = Floor-to-floor height, ft
k = Factor in bond strength calculation
l = Unbraced length of column, in.
l_e = Encased length of steel shape, in.
m = Cantilever distance in base plate analysis, in.
n = Cantilever distance in base plate analysis, in.
r = Radius of gyration, in.
r_m = Radius of gyration of steel shape in composite column, in.
s = Spacing (clear distance), in.
t_f = Flange thickness, in.
t_p = Thickness of base plate, in.
t_w = Web thickness, in.
w_c = Unit weight of concrete, lbs/ft^3
β_1 = Factor for determining depth of concrete in compression
Δ_{oh} = Translation deflection of story, in.
λ_c = Column slenderness parameter
ϕ_b = Resistance factor for flexure
ϕ_c = Resistance factor for axially loaded composite column

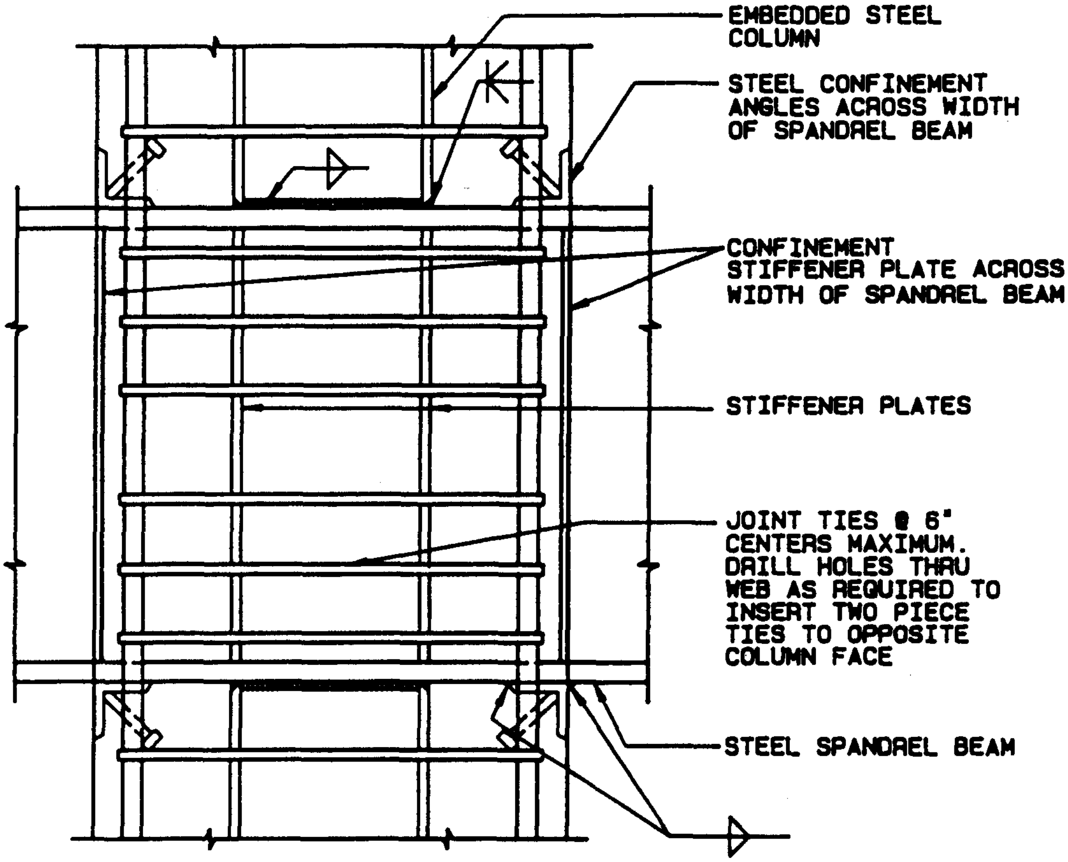
PART 2: SUGGESTED DETAILS FOR COMPOSITE COLUMNS



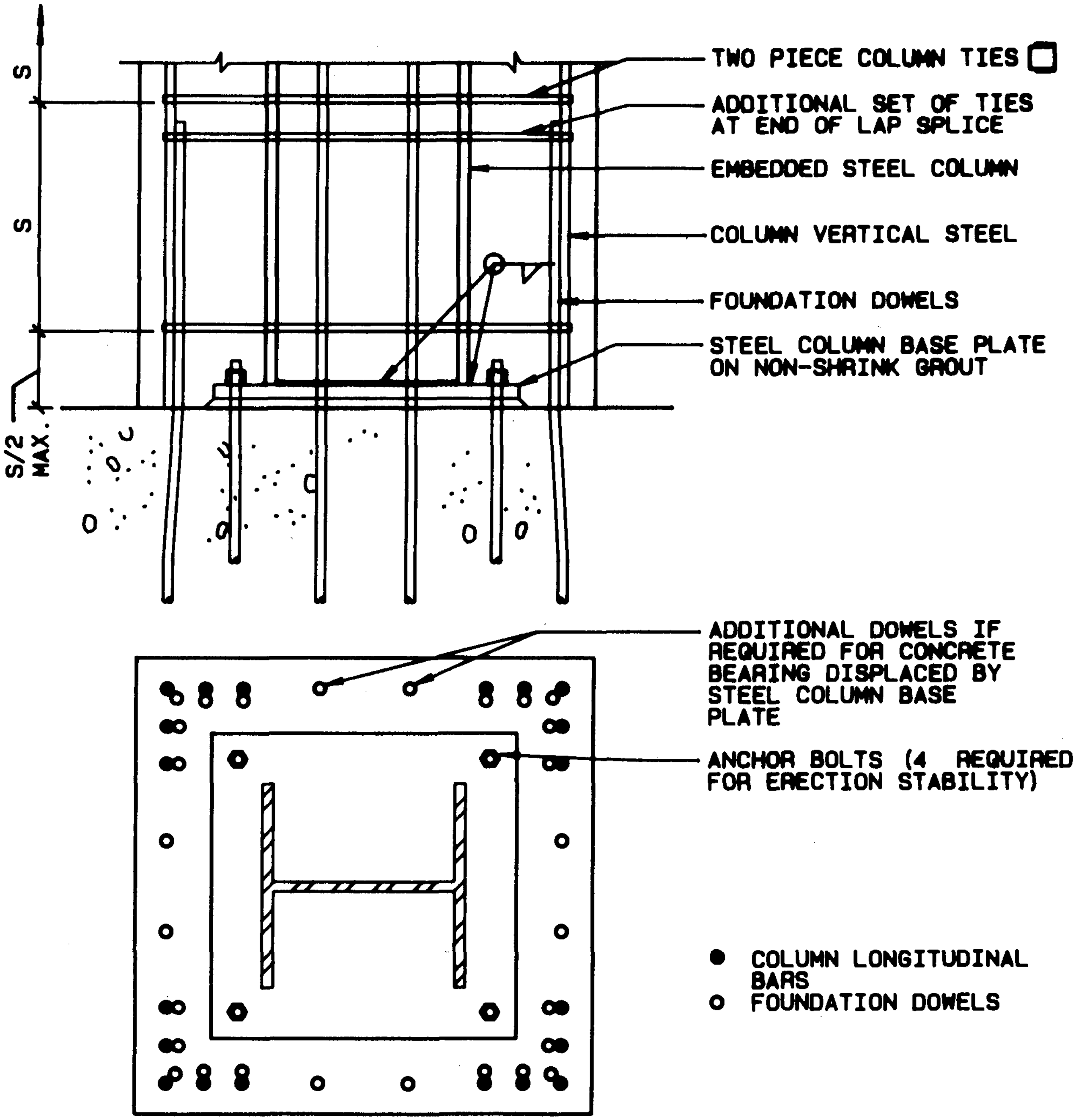
Typical Detail 1: Composite column elevation.



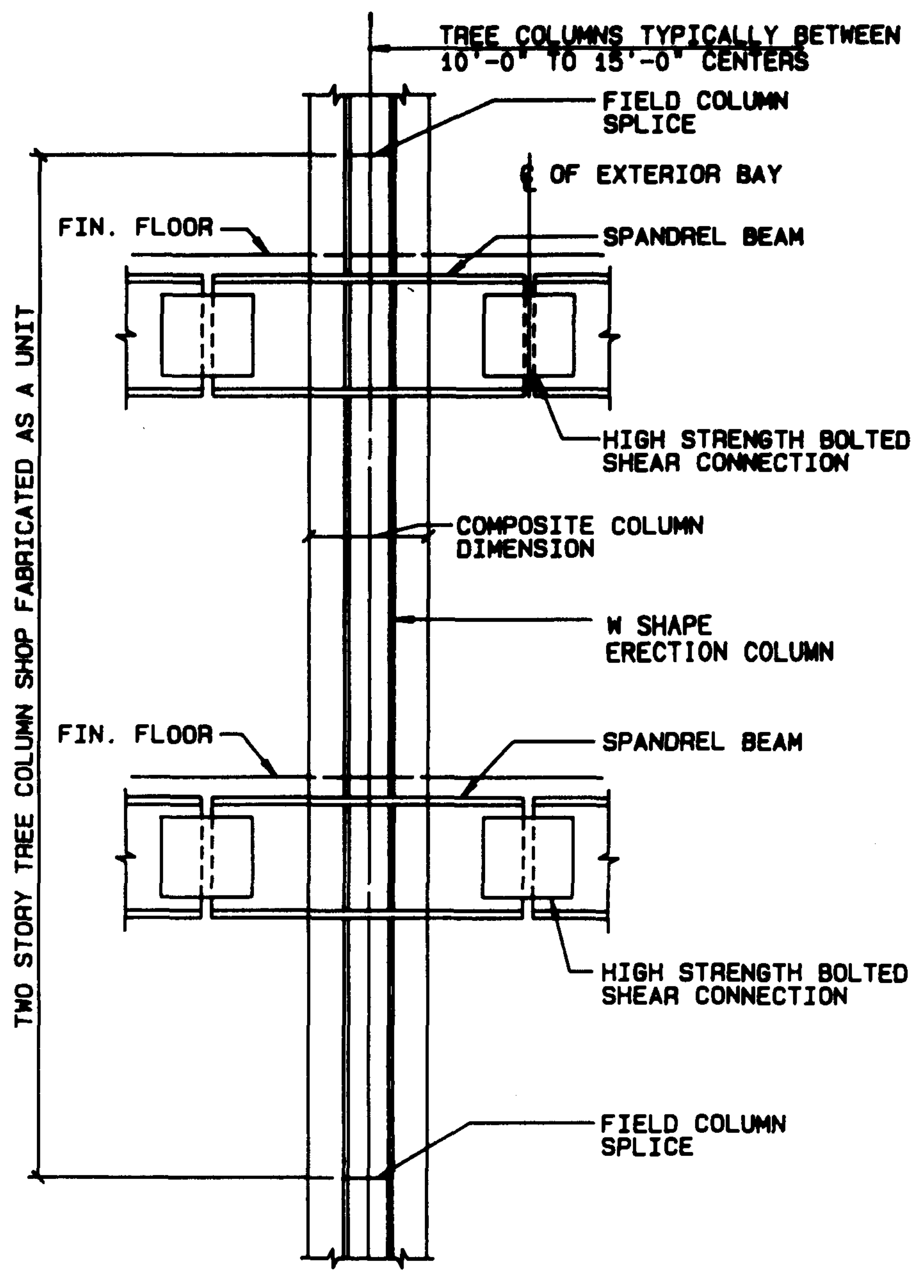
Typical Detail 2: Composite column cross section.



Typical Detail 3: Composite column joint.



Typical Detail 4: Composite column baseplate.



Typical Detail 5: Tree column in a composite frame.

PART 3: DESIGN EXAMPLES

Example 1:

Compute the axial load capacity of a 48×48-in. composite column with an encased W14×730. Compute capacity for unbraced length equal to 11'-0 and 40'-0. Use $f_c' = 5$ ksi, $F_{yr} = 60$ ksi, 20 - #14 (6x - 6y) and $w_c = 145$ pcf. See Fig. B-1.

W14×730 properties are:

$A_s = 215$ in.^{2}
$d = 22.42$ in.
$t_w = 3.07$ in.
$b_f = 17.89$ in.
$t_f = 4.91$ in.
$r_x = 8.17$ in.
$r_y = 4.69$ in.

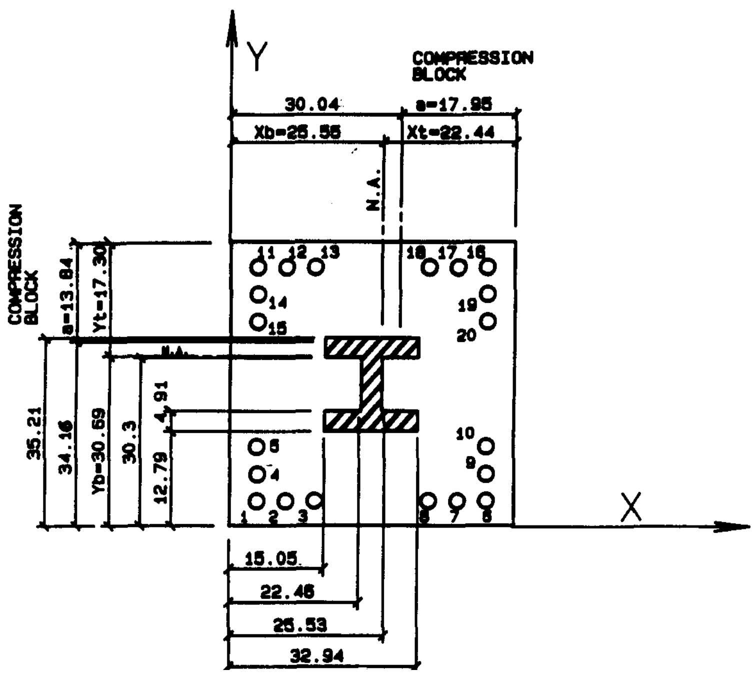


Fig. B-1. Cross section for Examples 1 and 2.

Solution:

1. Compute section properties.

Total area of longitudinal reinforcing bars = 20 × 2.25 = 45.0 in.^{2}

Gross section area of concrete column = 48 × 48 = 2,304 in.^{2}

Percentage of longitudinal reinforcing bars = 45.0 / 2,340 = 1.95 percent

Percentage of steel shape = 2157 2,304 = 9.33 percent > 4 percent **o.k.**

Net area of concrete = 2,304 - 45 - 215 = 2,044 in.^{2}

$$F_{my} = F_y + (c_1 \times F_{yr} \times A_r / A_s) + (c_2 \times f_c' \times A_c / A_s)$$

(Use F_{yr} = 60 ksi instead of 55 ksi limitations—see discussion under "Shear Connections")

$$= 50 + (0.7 \times 60 \times 45 / 215) + (0.6 \times 5 \times 2{,}044 / 215)$$

$$= 87.31 \text{ ksi}$$

$$E_c = w_c^{1.5} \times f_c'^{\,0.5} = 145^{1.5} \times 5^{0.5} = 3{,}904 \text{ ksi}$$

$$E_m = E_s + (c_3 \times E_c \times A_c / A_s)$$

$$= 29{,}000 + (0.2 \times 3{,}904 \times 2{,}044 / 215) = 36{,}423 \text{ ksi}$$

Table A

COMPOSITE BEAM—COLUMN DESIGN CAPACITY — LRFD

$\phi_c = 0.85$ f_c': 5.0 ksi NW
$\phi_b = 0.90$ F_{yr}: 60 ksi

Axial Load Capacity (kips), Uniaxial Moment Capacity (ft-kips) — Column Size (b x h): 48 x 48

Designation		W14x730								W14x665							
F_y (ksi)		36				50				36				50			
Reinf.	KL	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}
.54%	0	12300	0.0	8170	6960	14900	0.0	10100	7970	11800	0.0	7650	6680	14100	0.0	9370	7630
A_r (in.^{2})	11	12200	0.2	7350	6260	14800	0.2	9080	7170	11700	0.2	6880	6010	14000	0.2	8440	6860
= 12.48	13	12200	0.3	6430	5480	14700	0.3	7950	6270	11700	0.3	6020	5260	14000	0.3	7380	6010
	17	12100	0.4	5510	4700	14600	0.4	6810	5380	11600	0.4	5160	4510	13800	0.4	6330	5150
8-#11	21	12000	0.5	4590	3910	14500	0.5	5680	4480	11500	0.5	4300	3760	13700	0.5	5270	4290
$4x - 2y$	25	11900	0.7	2760	2350	14300	0.7	3410	2690	11400	0.7	2580	2260	13500	0.7	3160	2570
	40	11300	0.9	918	782	13400	0.9	1140	896	10800	0.9	860	751	12700	0.9	1050	857
#4 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@28 in		11200	11200	14.40	14.40	11200	11200	14.40	14.40	10400	10400	14.40	14.40	10400	10400	14.40	14.40
1.04%	0	12700	0.0	9110	7740	15300	0.0	11000	8750	12200	0.0	8590	7470	14500	0.0	10300	8410
A_r (in.^{2})	11	12600	0.2	8200	6970	15100	0.2	9930	7870	12100	0.2	7730	6720	14400	0.2	9280	7570
= 24.00	13	12600	0.3	7170	6090	15100	0.3	8690	6890	12100	0.3	6760	5880	14300	0.3	8120	6620
	17	12500	0.4	6150	5220	15000	0.4	7440	5900	12000	0.4	5800	5040	14200	0.4	6960	5670
24-#9	21	12400	0.5	5120	4350	14800	0.5	6200	4920	11900	0.5	4830	4200	14100	0.5	5800	4730
$8x - 6y$	25	12300	0.7	3070	2610	14600	0.7	3720	2950	11700	0.7	2900	2520	13900	0.7	3480	2840
	40	11600	0.9	1020	870	13700	0.9	1240	983	11100	0.9	966	840	13000	0.9	1160	945
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@15 in		11200	11200	14.40	14.40	11200	11200	14.40	14.40	10400	10400	14.40	14.40	10400	10400	14.40	14.40
1.95%	0	13400	0.0	10700	9550	16000	0.0	12600	10600	12900	0.0	10200	9280	15200	0.0	11900	10200
A_r (in.^{2})	11	13300	0.2	9620	8600	15800	0.2	11300	9500	12800	0.2	9150	8350	15100	0.2	10700	9190
= 45.00	13	13300	0.3	8420	7520	15800	0.3	9930	8310	12700	0.3	8010	7310	15000	0.3	9370	8040
	17	13200	0.4	7220	6450	15600	0.4	8510	7120	12600	0.4	6860	6260	14900	0.4	8030	6900
20-#14	21	13100	0.5	6010	5370	15500	0.5	7090	5940	12500	0.5	5720	5220	14700	0.5	6690	5750
$6x - 6y$	25	12900	0.7	3610	3220	15300	0.7	4260	3560	12400	0.7	3430	3130	14500	0.7	4010	3450
	40	12200	0.9	1200	1070	14300	0.9	1420	1190	11700	0.9	1140	1040	13600	0.9	1340	1150
#4 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@28 in		11100	11100	14.40	14.40	11100	11100	14.40	14.40	10400	10400	14.40	14.40	10400	10400	14.40	14.40
2.78%	0	14000	0.0	12500	10500	16600	0.0	14400	11500	13500	0.0	12000	10300	15800	0.0	13700	11200
A_r (in.^{2})	11	13900	0.2	11200	9490	16400	0.2	13000	10400	13400	0.2	10800	9250	15700	0.2	12300	10100
= 64.00	13	13900	0.3	9830	8310	16400	0.3	11300	9090	13300	0.3	9420	8090	15600	0.3	10800	8830
	17	13800	0.4	8430	7120	16200	0.4	9720	7790	13200	0.4	8080	6940	15500	0.4	9240	7570
16-#18	21	13600	0.5	7020	5930	16100	0.5	8100	6490	13100	0.5	6730	5780	15300	0.5	7700	6310
$6x - 4y$	25	13500	0.7	4210	3560	15800	0.7	4860	3900	13000	0.7	4040	3470	15100	0.7	4620	3780
	40	12700	0.9	1400	1190	14700	0.9	1620	1300	12200	0.9	1350	1160	14000	0.9	1540	1260
#4 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
(5)28 in		11100	11100	14.40	14.40	11100	11100	14.40	14.40	10300	10300	14.40	14.40	10300	10300	14.40	14.40
4.17%	0	15100	0.0	14600	12300	17600	0.0	16600	13300	14600	0.0	14100	12000	16900	0.0	15800	12900
A_r (in.^{2})	11	15000	0.2	13200	11100	17500	0.2	14900	12000	14400	0.2	12700	10800	16700	0.2	14300	11700
= 96.00	13	14900	0.3	11500	9680	17400	0.3	13000	10500	14400	0.3	11100	9470	16700	0.3	12500	10200
	17	14800	0.4	9880	8300	17300	0.4	11200	8970	14300	0.4	9530	8110	16500	0.4	10700	8740
24-#18	21	14700	0.5	8240	6910	17100	0.5	9310	7470	14100	0.5	7940	6760	16300	0.5	8910	7280
$8x - 6y$	25	14500	0.7	4940	4150	16800	0.7	5590	4480	13900	0.7	4770	4060	16100	0.7	5350	4370
	40	13600	0.9	1650	1380	15600	0.9	1860	1490	13000	0.9	1590	1350	14800	0.9	1780	1460
#4 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@28 in		11100	11100	14.40	14.40	11100	11100	14.40	14.40	10300	10300	14.40	14.40	10300	10300	14.40	14.40

Notes: 1. $C_{ex} = P_{ex}(K_xL_x)^2/10{,}000$ (kip-ft^2), $C_{ey} = P_{ey}(K_yL_y)^2/10{,}000$ (kip-ft^2), KL in ft, r_{mx} and r_{my} in inches.

2. Zeroes in columns for $\phi_c P_n$, M_{ux}, and M_{uy} indicate that no suitable reinforcing bar arrangement is available for the indicated steel percentage.

3. See Figure 2 lor definition of bar arrangement (roc - my). NW = normal weight concrete.

4. $M_{ux} = \phi_b M_{nx}$ and $M_{uy} = \phi_b M_{ny}$ when $P_u/(\phi_c P_n) = 0.0$

$r_{mx} = r_{my} = 0.3 \times 48 = 14.4$ in. $> r_x = 8.17$ in.

$$C_{ex} = C_{ey} = P_{ex} \times KL^2 / 10{,}000 = A_s \times E_m(\pi r_{mx})^2 / 1{,}440{,}000$$

$$= 215 \times 36{,}423 \times (3.1416 \times 14.4)^2 / 1{,}440{,}000$$

$$= 11{,}130 \text{ kip-ft}^2$$

2. Axial load capacity

For KL = 0'-0

$F_{cr} = F_{my} = 87.31$ ksi

$\phi P_n = \phi A_s F_{cr} = 0.85 \times 215 \times 87.31 = 15{,}960$ kips

For KL=11'-0

$$\lambda_c = KL(F_{my} / E_m)^{0.5} / r_m / \pi$$

$$= KL(87.31 / 36.423)^{0.5} / 14.4 / 3.1416 = 0.001082KL$$

$$= 11 \times 12 \times 0.001082 = 0.143 < 1.5$$

$F_{cr} = 0.658^{\lambda_c^2} \times F_{my} = 0.658^{(0.143 \times 0.143)} \times 87.31 = 86.57$ ksi

$\phi P_n = \phi A_s F_{cr} = 0.85 \times 215 \times 86.57 = 15{,}820$ kips

For KL = 40'-0

$\lambda_c = 0.001082KL = 40 \times 12 \times 0.001082 = 0.520 < 1.5$

$F_{cr} = 0.658^{(0.520 \times 0.520)} \times 87.31 = 78.0$ ksi

$\phi P_n = \phi A_s F_{cr} = 0.85 \times 215 \times 78.0 = 14{,}250$ kips

The calculated values of ϕP_n agree with the values circled in Table A, Example 2, which have been rounded.

Example 2:

Compute the interaction curves of the composite column described in Example 1. See Fig. B-1.

Solution:

1. Coordinates of reinforcing bars.

No.	x	y	No.	x	y
1	2.846	2.846	11	2.846	45.154
2	7.079	2.846	12	7.079	45.154
3	11.312	2.846	13	11.312	45.154
4	2.846	7.079	14	2.846	40.921
5	2.846	11.312	15	2.846	36.688
6	45.154	2.846	16	45.154	45.154
7	40.921	2.846	17	40.921	45.154
8	36.688	2.846	18	36.688	45.154
9	45.154	7.079	19	45.154	40.921
10	45.154	11.312	20	45.154	36.688

2. Nominal flexural strength about x-axis.

$$fc85 = 0.85f_c' = 0.85 \times 5 = 4.25$$

$$Fy85 = F_y - fc85 = 50 - 4.25 = 45.75$$

$$Fyr85 = F_{yr} - fc85 = 60 - 4.25 = 55.75$$

$$\beta_1 = 0.85 - 0.05\,(f_c' - 4) = 0.80$$

In general, successive approximations are required to determine the location of the plastic neutral axis. Here, trial values of the distance from the plastic neutral axis to the bottom of the section, Y_b, and to the top of the section, Y_a, are assumed as follows:

$$Y_b = 30.69 \text{ in.}$$

$$Y_t = 48 - Y_b = 48 - 30.69 = 17.31 \text{ in.}$$

$$a = \beta_1 Y_t = 0.8 \times 17.31 = 13.85 \text{ in.}$$

$$Y_a = 48 - a = 48 - 13.85 = 34.15 \text{ in.}$$

	Force (kips)	y-Y_b (in.)	Moment (ft-kips)
Concrete			
4.25 × 48 × 13.8445	2824.28	10.3834	2443.80
Rebars			
1. -60 × 2.25	-135.0	-27.8484	313.29
2. -60 × 2.25	-135.0	-27.8484	313.29
3. -60 × 2.25	-135.0	-27.8484	313.29
4. -60 × 2.25	-135.0	-23.6154	265.67
5. -60 × 2.25	-135.0	-19.3824	218.05
6. -60 × 2.25	-135.0	-27.8484	313.29
7. -60 × 2.25	-135.0	-27.8484	313.29
8. -60 × 2.25	-135.0	-27.8484	313.29
9. -60 × 2.25	-135.0	-23.6154	265.67
10. -60 × 2.25	-135.0	-19.3824	218.05
11. 55.75 × 2.25	125.4375	14.4596	151.15
12. 55.75 × 2.25	125.4375	14.4596	151.15
13. 55.75 × 2.25	125.4375	14.4596	151.15
14. 55.75 × 2.25	125.4375	10.2266	106.90
15. 55.75 × 2.25	125.4375	5.9936	62.65
16. 55.75 × 2.25	125.4375	14.4596	151.15
17. 55.75 × 2.25	125.4375	14.4596	151.15
18. 55.75 × 2.25	125.4375	14.4596	151.15
19. 55.75 × 2.25	125.4375	10.2266	106.90
20. 55.75 × 2.25	125.4375	5.9936	62.65
Subtotal	-95.625		4093.18
Steel			
(50 - 0.85 × 5)(35.21 - 34.1555) × 17.89	863.07	3.9884	286.86
50 × (34.1555 - 30.6944) × 17.89	3095.95	1.7306	446.49
-50 × (30.6944 - 30.3) × 17.89	-352.79	-0.1972	5.80
-50 × (30.3-17.7) × 3.07	-1934.10	-6.6944	1078.97
-50 × 4.91 × 17.89	-4392.00	-15.4494	5654.47
Subtotal	-2728.87		7472.59
Total	-0.22		14009

Since the summation of forces is approximately zero, the assumed location of the plastic neutral axis is correct.

$$\phi M_{nx} = 0.9 \times 14{,}009 = 12{,}608 \text{ kip-ft}$$

Calculate the uniaxial moment capacity from Eqs. H1-1a and H1-1b for assumed values of the load ratio $R_u = P_u / \phi_c P_n$.

$$M_{ux} = 12{,}608 \times (1 - R_u) \times 9/8 \quad \text{if } R_u \geq 0.2$$

$$M_{ux} = 12{,}608 \times (1 - R_u / 2.0) \quad \text{if } R_u < 0.2$$

Points on the interaction curve are calculated as follows:

R_u	0.0	0.2	0.3	0.4	0.5	0.7	0.9
M_{ux}	12,610	11,350	9,930	8,510	7,090	4,260	1,420

These values agree with the circled values in Table A.

3. Nominal flexural strength about y-axis.

Try

$$X_b = 25.55 \text{ in.}$$

$$X_t = 48 - X_b = 48 - 25.55 = 22.45 \text{ in.}$$

$a = \beta_1 X_t = 0.8 \times 22.45 = 17.96$ in.

$X_a = 48 - a = 48 - 17.96 = 30.04$ in.

	Force (kips)	x-X_b (in.)	Moment (ft-kips)
Concrete			
4.25 × 48 × 17.9565	3663.13	13.4674	4111.07
Rebars			
1. -60 × 2.25	-135.0	-22.7084	255.47
2. -60 × 2.25	-135.0	-18.4754	207.85
3. -60 × 2.25	-135.0	-14.2424	160.23
4. -60 × 2.25	-135.0	-22.7084	255.47
5. -60 × 2.25	-135.0	-22.7084	255.47
6. 55.75 × 2.25	125.4375	19.5996	204.88
7. 55.75 × 2.25	125.4375	15.3666	160.63
8. 55.75 × 2.25	125.4375	11.1336	116.38
9. 55.75 × 2.25	125.4375	19.5996	204.88
10. 55.75 × 2.25	125.4375	19.5996	204.88
11. -60 × 2.25	-135.0	-22.7084	255.47
12. -60 × 2.25	-135.0	-18.4754	207.85
13. -60 × 2.25	-135.0	-14.2424	160.23
14. -60 × 2.25	-135.0	-22.7084	255.47
15. -60 × 2.25	-135.0	-22.7084	255.47
16. 55.75 × 2.25	125.4375	19.5996	204.88
17. 55.75 × 2.25	125.4375	15.3666	160.63
18. 55.75 × 2.25	125.4375	11.1336	116.38
19. 55.75 × 2.25	125.4375	19.5996	204.88
20. 55.75 × 2.25	125.4375	19.5996	204.88
Subtotal (Rebars)	-95.625		4052.28
Steel			
(50 - 0.85 × 5)(32.945 - 30.0435) × 4.91 × 2	1303.54	5.9399	645.24
50 × (30.0435 - 25.5544) × 4.91 × 2	2204.15	2.2446	412.29
-50 × (25.5544 - 25.535) × 4.91 × 2	-9.53	-0.0097	0.01
-50 × 3.07 × 22.42	-3441.47	-1.5544	445.79
-50 × 7.41 × 4.91 × 2	-3638.31	-6.7944	2060.01
Subtotal (Steel)	-3581.62		3563.34
Total	-14.12		11730

$\phi M_{nx} = 0.9 \times 11{,}730 = 10{,}550$ kip-ft

$M_{ux} = 10{,}550 \times (1.0 - R_u / 2.0)$ if $R_u = P_u / \phi_c P_n \le 0.2$

$M_{ux} = 10{,}550 \times (1.0 - R_u) \times 9/8$ if $R_u = P_u / \phi_c P_n > 0.2$

R_u	0.0	0.2	0.3	0.4	0.5	0.7	0.9
M_{ux}	10,550	9,500	8,310	7,120	5,940	3,560	1,190

These values agree with the circled values in Table A.

Example 3:

Design a 20×20-in. composite column with an encased W-shape to resist a factored axial load of 470 kips and a factored moment about the x-axis of 350 kip-ft. The loads are obtained from a second order analysis. Use $f_c' = 5$ ksi, $F_{yr} = 60$ ksi, $F_y = 50$ ksi, and $KL = 17$ ft.

Solution:

1. Calculate relative eccentricity:

   $M_u / (P_u t) = 350 / (470 \times 1.67) = 0.45$

2. Determine trial load ratio:

   $M_u / (P_u t) > 0.33$, use $R_u = 0.3$

3. Calculate required axial strength:

   $\phi_c P_n = P_u / R_u = 470 / 0.3 = 1{,}567$ kips

4. Select trial column:

   Try 20×20-in. composite column, W8×58 column, 4-#7 (2*x* - 2*y*)

   $\phi_c P_n = 1{,}570$ kips for $KL = 17$ ft

5. Calculate load ratio for trial column:

   $R_u = P_u / \phi_c P_n = 470 / 1{,}570 = 0.33$

6. Determine uniaxial moment capacity:

   From Table B with $R_u = P_u / \phi_c P_n = 0.3$, $M_{ux} = 354$ kip-ft

7. Compare to factored moment:

   $M_{ux} = 354$ kip-ft (from Table B) > 350 kip-ft required **o.k.**

   Use 20×20-in. composite column with W8×58 ($F_y = 50$ ksi), $f_c' = 5$ ksi, 4-#7 bars (2*x* - 2*y*) vertical bars and #3 ties at 13 in.

Table B

COMPOSITE BEAM—COLUMN DESIGN CAPACITY — LRFD
(See Examples 3 and 4)

$\phi_c = 0.85$ f_c': 5.0 ksi NW
$\phi_b = 0.90$ F_{yr}: 60 ksi

Axial Load Capacity (kips), Uniaxial Moment Capacity (ft-kips)

Column Size (b x h): 48 x 48

Designation		W8x67								W8x58							
F_y (ksi)		36				50				36				50			
Reinf.	KL	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}	$\phi_c P_n$	$P_u/(\phi_c P_n)$	M_{ux}	M_{uy}
.60%	0	1650	0.0	408	397	1890	0.0	492	455	1580	0.0	377	374	1780	0.0	450	429
A_r (in.^{2})	11	1580	0.2	367	357	1790	0.2	443	410	1510	0.2	340	336	1690	0.2	405	386
= 2.40	13	1550	0.3	321	313	1750	0.3	388	358	1480	0.3	297	294	1650	0.3	354	338
	17	1480	0.4	275	268	1660	0.4	332	307	1410	0.4	255	252	1570	0.4	304	289
4-#7	21	1400	0.5	229	223	1560	0.5	277	256	1330	0.5	212	210	1460	0.5	253	241
$2x-2y$	25	1300	0.7	137	134	1440	0.7	166	153	1230	0.7	127	126	1350	0.7	152	144
	40	898	0.9	45	44	941	0.9	55	51	836	0.9	42	42	869	0.9	50	48
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@13 in		213	213	6.00	6.00	213	213	6.00	6.00	195	195	6.00	6.00	195	195	6.00	6.00
1.00%	0	1710	0.0	460	449	1940	0.0	544	507	1630	0.0	430	426	1840	0.0	502	481
A_r (in.^{2})	11	1630	0.2	414	404	1840	0.2	490	456	1550	0.2	387	383	1740	0.2	452	432
= 4.00	13	1600	0.3	362	354	1800	0.3	429	399	1520	0.3	338	335	1700	0.3	395	378
	17	1520	0.4	311	303	1700	0.4	367	342	1450	0.4	290	287	1610	0.4	339	324
4-#9	21	1430	0.5	259	252	1590	0.5	306	285	1360	0.5	241	239	1500	0.5	282	270
$2x-2y$	25	1330	0.7	155	151	1470	0.7	183	171	1260	0.7	145	143	1370	0.7	169	162
	40	908	0.9	51	50	948	0.9	61	57	845	0.9	48	47	875	0.9	56	54
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@13 in		213	213	6.00	6.00	213	213	6.00	6.00	195	195	6.00	6.00	195	195	6.00	6.00
2.00%	0	1840	0.0	594	514	2070	0.0	678	571	1760	0.0	563	491	1970	0.0	636	546
A_r (in.^{2})	11	1750	0.2	535	463	1960	0.2	610	514	1670	0.2	507	442	1850	0.2	572	491
= 8.00	13	1710	0.3	468	405	1910	0.3	534	450	1640	0.3	444	386	1810	0.3	501	430
	17	1630	0.4	401	347	1800	0.4	458	385	1550	0.4	380	331	1700	0.4	429	368
8-#9	21	1520	0.5	334	289	1680	0.5	381	321	1450	0.5	317	276	1580	0.5	358	307
$4x-2y$	25	1410	0.7	200	173	1540	0.7	229	192	1340	0.7	190	165	1440	0.7	214	184
	40	930	0.9	66	57	961	0.9	76	64	863	0.9	63	55	887	0.9	71	61
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@13 in		212	212	6.00	6.00	212	212	6.00	6.00	194	194	6.00	6.00	194	194	6.00	6.00
3.00%	0	1970	0.0	679	647	2200	0.0	764	704	1900	0.0	649	624	2100	0.0	721	679
A_r (in.^{2})	11	1860	0.2	611	583	2070	0.2	687	633	1790	0.2	584	562	1970	0.2	649	611
= 12.00	13	1820	0.3	535	510	2020	0.3	601	554	1750	0.3	511	492	1920	0.3	568	534
	17	1730	0.4	458	437	1900	0.4	515	475	1650	0.4	438	421	1800	0.4	487	458
12-#9	21	1610	0.5	382	364	1760	0.5	429	396	1530	0.5	365	351	1660	0.5	406	381
$4x-4y$	25	1480	0.7	229	218	1600	0.7	257	237	1400	0.7	219	210	1500	0.7	243	229
	40	947	0.9	76	72	971	0.9	85	79	877	0.9	73	70	894	0.9	81	76
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@13 in		211	211	6.00	6.00	211	211	6.00	6.00	193	193	6.00	6.00	193	193	6.00	6.00
3.81%	0	2080	0.0	762	721	2310	0.0	846	778	2000	0.0	731	698	2210	0.0	804	752
A_r (in.^{2})	11	1960	0.2	686	649	2170	0.2	761	700	1880	0.2	658	628	2060	0.2	723	676
= 15.24	13	1910	0.3	600	568	2110	0.3	666	613	1840	0.3	576	549	2010	0.3	633	592
	17	1810	0.4	514	486	1980	0.4	571	525	1730	0.4	494	471	1880	0.4	542	507
12-#10	21	1680	0.5	428	405	1820	0.5	476	437	1600	0.5	411	392	1720	0.5	452	423
$4x-4y$	25	1540	0.7	257	243	1650	0.7	285	262	1460	0.7	247	235	1550	0.7	271	253
	40	957	0.9	85	81	976	0.9	95	87	885	0.9	82	78	897	0.9	90	84
#3 Ties		C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}	C_{ex}	C_{ey}	r_{mx}	r_{my}
@13 in		211	211	6.00	6.00	211	211	6.00	6.00	193	193	6.00	6.00	193	193	6.00	6.00

Notes: 1. $C_{ex} = P_{ex}(K_xL_x)^2/10{,}000$ (kip-ft^2), $C_{ey} = P_{ey}(K_yL_y)^2/10{,}000$ (kip-ft^2), in ft, r_{mx} and r_{my} in inches.

2. Zeroes in columns lor $\phi_c P_n$, M_{ux}, and M_{uy} indicate that no suitable reinforcing bar arrangement is available for the indicated steel percentage.

3. See Figure 2 for definition of bar arrangement *(nx- my).* NW = normal weight concrete.

4. $M_{ux} = \phi_b M_{nx}$ and $M_{uy} = \phi_b M_{ny}$ when $P_u/(\phi_c P_n) = 0.0$

Example 4:

Design a 20×20-in. composite column with an encased W-shape to resist a factored axial load of 1,190 kips and a factored moment about the x-axis of 180 kip-ft. The loads are obtained from a second order analysis. Use $f_c' = 5$ ksi, $F_{yr} = 60$ ksi, $F_y = 50$ ksi, and $KL = 17$ ft.

Solution:

1. Calculate relative eccentricity:

   $M_u / (P_u t) = 180 / (1{,}190 \times 1.67) = 0.09$

2. Determine trial load ratio:

   $M_u / (P_u t) < 0.10$, use $R_u = 0.7$

3. Calculate required axial strength:

   $\phi_c P_n = P_u / R_u = 1{,}190 / 0.7 = 1{,}700$ kips

4. Select trial column:

   Try 20×20-in. composite column, W8×67 column, 4-#9 (2x - 2y)

   $\phi_c P_n = 1{,}700$ kips for $KL = 17$ ft

5. Calculate load ratio for trial column:

   $R_u = P_u / \phi_c P_n = 1{,}190 / 1{,}700 = 0.7$

6. Determine uniaxial moment capacity:

   From Table B with $R_u = P_u / \phi_c P_n = 0.7$, $M_{ux} = 183$ kip-ft

7. Compare to factored moment:

   $M_{ux} = 183$ kip-ft (from Table B) > 180 kip-ft required **o.k.**

   Use 20×20-in. composite column with W8×67 ($F_y = 50$ ksi), $f_c' = 5$ ksi, 4-#9 bars (2x - 2y) vertical bars and #3 ties at 13 in.

Example 5:

Design the base plate of a 18×18-in. composite column with an encased W10×54 of F_y = 36 ksi, f_c'= 8 ksi, and 4-#8 grade 60 longitudinal bars. Factored axial load P_u = 1,000 kips, KL = 31 ft. Use f_c'= 3 ksi for footing. Assume $(A_2 / A_1)^{1/2} \geq 2$.See Fig. B-2 for nomenclature. Refer to AISC LRFD Manual, p. 2-101 for base plate design procedure.

Solution:

Base plate will be designed for the portion of the factored axial load resisted by the W10×54.

W10×54 properties:

b_f = 10.03 in.
d = 10.09 in.
t_f = 0.615 in.
A_s = 15.8 in.^{2}

Try base plate 12×12 in.

1. Compute axial load carried by W10×54 based on the contribution of W10×54 to the total column capacity.

$$F_{my} = F_y + F_{yr}\ (A_r / A_s) + c_2 f_c'\ (A_c / A_s)$$

$$= 36 + (0.7 \times 60 \times 3.16 / 15.8) + (0.6 \times 8(18 \times 18 - 15.8 - 4 \times 0.79) / 15.8)$$

$$= 137.07 \text{ ksi}$$

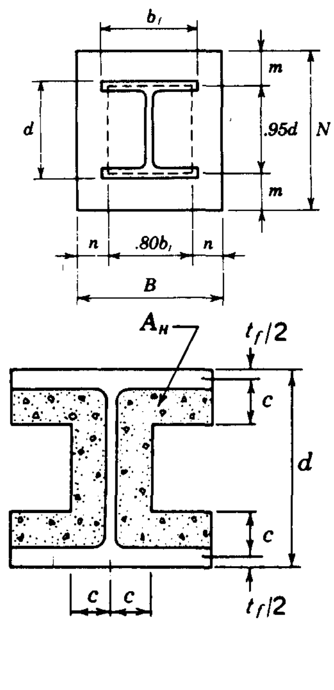
Portion of factored axial load resisted by W10×54 is:

$$P_s = 1{,}000 \times 36 / 137.07 = 262.64 \text{ kips}$$

2. Compute m and n.

$$m = (N - 0.95d) / 2 = (12 - 0.95 \times 10.09) / 2 = 1.207$$

$$n = (B - 0.8b_f) / 2 = (12 - 0.8 \times 10.03) / 2 = 1.988 \quad \textbf{governs}$$



P_o = factored load contributory to area enclosed by steel shape, kips
P_s = Factored axial load resisted by steel shape, kips
A_1 = Area of base plate, in.^{2}
A_2 = Full cross sectional area of concrete support, in.^{2}
A_H = Area of H-shaped portion of base plate in light columns, in.^{2}
F_y = Specified minimum yield stress of steel, ksi
f_c' = Specified compressive strength of concrete, ksi
t_p = Thickness of base plate, in.
ϕ_c = Resistance factor for concrete = 0.6
ϕ_p = Resistance factor for base plate = 0.9

Fig. B-2. Column base plates.

3. Concrete bearing stress.

$\phi 0.85 f_c' \, (A_2 / A_1)^{0.5} = 0.6 \times 0.85 \times 3 \times 2 = 3.06$ ksi

4. Check concrete bearing under base plate.

$P_s / (BN) = 262.64 / (12 \times 12) = 1.824$ ksi < 3.06 **o.k.**

5. Compute factored load contributary to the area enclosed by W10×54.

$P_o = P_s \, b_f d / (BN) = 262.64 \times 10.03 \times 10.09 / (12 \times 12) = 184.58$ **kips**

6. Compute area of H-shaped region.

$A_H = P_o / (0.6 \times 1.7 \times f_c') = 184.58 / (0.6 \times 1.7 \times 3) = 60.32$ in.^{2}

7. Compute *c*.

$c = (d + b_f - t_f - ((d + b_f - t_f)^2 - 4(A_H - t_f b_f))^{1/2}) / 4$

$= (10.09 + 10.03 - 0.615 - ((10.09 + 10.03 - 0.615)^2 - 4 \times (60.32 - 0.615 \times 10.03))^{1/2}) / 4$

$= 1.676$

8. Compute base plate thickness.

$t_p = \max(m, n) \times (2P_s \, / (0.9F_y BN))^{1/2}$

$= 1.988 \times (2 \times 262.64 / (0.9 \times 36 \times 12 \times 12))^{1/2} = 0.667$ in.

$t_p = c(2P_o \, / 0.9F_y A_H))^{1/2}$

$= 1.676 \times (2 \times 184.58 / (0.9 \times 36 \times 60.32))^{1/2} = 0.728$ in.

Use ¾-in. plate.

9. Design dowels to foundation.

Allowable compression transfer by concrete:

$= 2\phi_c 0.85 f_c'$ (*column area – base plate area*)

$= 2 \times 0.6 \times 0.85 \times 3 \, (18 \times 18 - 12 \times 12)$

$= 550.8$ **kips**

Required compression transfer by concrete:

$= 1{,}000 - 262.64$

$= 737.36$ kips > 550.8 kips **Dowels are required.**

Required area of dowels:

A_d (req'd) $= (737.36 - 550.8) / 60 = 3.11$ in.^{2}

A_d (min.) $= 0.005A_g = 0.005 \times 18 \times 18 = 1.62$ in.^{2} (ACI 318-89 Section 15.8.2.1)

Use 4-#8, A_s *(provided)* $= 4 \times 0.79 = 3.16$ in.^{2} > 3.11 **o.k.**

Embed dowels 22 bar diameters (for 3,000 psi concrete) into foundation (ACI 318-89 Section 12.3.1) = 22 × 1.00 = 22 in.

Dowel projection into column = 30 bar diameters (ACI 318-89 Section 12.16.1) = 30 × 1.00 = 30 in.

PROGRAM CMPOL

A computer program named CMPOL has been developed to generate composite column design tables as described in Part 4. The program may be used to generate the tables in either LRFD or ASD format. It is available through AISC. For further information and/or to place your software order, call (312) 670-2400.

The program is contained on a high quality 5¼-in. diskette or 3½-in. disk in executable form and may be copied to a hard disk. It will run on any IBM compatible computer (PC/XT/AT 286 or 386 or Model PS/2) with at least 512K installed RAM. A math coprocessor is optional.

The input data for CMPOL is all interactive. The procedure for running the program is as follows:

1. Access drive A or go to the subdirectory containing the program if it is located on the hard disk.
2. Set the printer in a condensed mode. This can be done by typing CONDENSE to invoke a batch file named RCOMP.BAT. The batch file automatically uncondenses the printer after the printing is finished.
3. Type CMPOL. A heading will appear on the screen followed by a question as to where the output is to be directed. Enter 2 for printer.
4. Questions will appear on the screen prompting the user to enter the following data:
   - Design Method. Enter 1 for LRFD, 2 for the approximate procedure as used in the LRFD Manual and described in the text to this guide, 3 for ASD.
   - Unbraced Length (ft). Input 7 values of unbraced length desired.
   - Vertical Reinforcing Bar Splice Type. (1 = bearing or mechanical butt splice, 2 = normal lap splice, 3 = tangential lap splice.) This selection impacts the bar positioning for clearance and cover checks.
   - Column Width (in.). Input minimum, maximum, and increment of column width. If the minimum and maximum are equal, then enter the increment as 1 to avoid an error.
   - Column Depth (in.). Input minimum, maximum, and increment of column depth as described above for column width.
   - Concrete strength (ksi). Input minimum, maximum, and increment of 28 day concrete strength f_c' entering the increment of 1 if the minimum and maximum are equal.
   - Concrete Unit Weight (pcf). This value is used in the determination of the modulus of elasticity for concrete.
   - Clear cover to reinforcing steel (in.). Input clear cover to reinforcing steel each direction and clear cover to rolled shape. All three values will normally be 1.5 in.
   - Reinforcing Steel Yield Strength (ksi).
   - Reinforcing Steel Size (integer number). Input minimum and maximum size of vertical reinforcing bars desired.
   - Reinforcing Steel Ratio (decimal number, i.e., 0.01). Input five percentages of reinforcing steel to be analyzed (typically 0.005, 0.01, 0.02, 0.03, 0.04).
   - Beam Clearance, Each Direction (in.). This number defines the clearance at the centerline of the column in each direction which is to be kept clear of vertical bars so that a beam may frame to the embedded rolled shape.
   - Embedded WF Shape. Nominal Depth (in.) Weight (PLF). Input the minimum and maximum W shape size to be included in the tables.
5. Tabular output will be sent to the printer and will be as shown in Appendices C and D. Note that some printers may not print the character "phi" (ϕ) in which case it will appear as an "m." A sample input screen and output are shown on the following pages.

```
Design method (1 = LRFD exact, 2 = LRFD approximate, 3 = ASD)? - - - - - - - - - 1
Input 7 unbraced length (ft) to be analyzed?- - - - - - - - - - - - - - - - - - - - - - - - - 0 11 13 17 21 25 40
Type of splice (1 = bearing, 2 = normal lap, 3 = tangential lap)? - - - - - - - - - - - - 3
Input min., max. & increment of column width (in)? - - - - - - - - - - - - - - - - - - - - 32 32 1
Input min., max. & increment of column depth (in)?- - - - - - - - - - - - - - - - - - - - - 32 32 1
Input min., max. & increment of concrete fc (ksi)? - - - - - - - - - - - - - - - - - - - - - - 3 3 1
Input unit weight of concrete (pcf)? - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - 145
Input clear cover CRx, CRy, CRw (in)?- - - - - - - - - - - - - - - - - - - - - - - - - - - - -  1.5 1.5 1.5
Input fy (ksi) of reinforcing steel? - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -  60
Input min., max. size of reinforcing steel?- - - - - - - - - - - - - - - - - - - - - - - - - - 7 18
Input 5 percentages of reinf. steel to be analyzed? - - - - - - - - - - - - - - - - - - - - .005 .01 .02 .03 .04
Input beam clearance reqd. in N-S & E-W dir. (in)? - - - - - - - - - - - - - - - - - - - -  5.5 5.5
Input min., max W shape (ND1, ND2, NW2)?-- - - - - - - - - - - - - - - - - - - - - - - 12 152 12 170

Do you want to run CMPOL again (1 = yes, 0 = no)?- - - - - - - - - - - - - - - - - - - -  0

Stop—program terminated.

C: I READY I CMPOL
```

ϕc = 0.85 f'c : 3.0 ksi NW
ϕb = 0.90 Fyr : 60 ksi

Axial Load Capacity (kips). Uniaxial Moment Capacity (ft-kips)
Column Size(b x h): 18 x 18

Designation		W8x67								W8x58							
Fy (ksi)		36				50				36				50			
Reinf.	KL	ϕcPn	$Pu/(\phi cPn)$	Mux	Muy	ϕcPn	$Pu/(\phi cPn)$	Mux	Muy	ϕcPn	$Pu/(\phi cPn)$	Mux	Muy	ϕcPn	$Pu/(\phi cPn)$	Mux	Muy
.74 X	0	1150	0.0	328	279	1380	0.0	409	319	1070	0.0	298	263	1280	0.0	367	300
Ar(in^2)		1100	0.2	295	251	1310	0.2	368	287	1030	0.2	268	237	1210	0.2	331	270
= 2.40	13	1080	0.3	258	219	1280	0.3	322	251	1010	0.3	234	207	1180	0.3	289	236
	17	1030	0.4	221	188	1220	0.4	276	215	960	0.4	201	177	1120	0.4	248	202
4-# 7	21	974	0.5	184	157	1130	0.5	230	179	905	0.5	167	148	1040	0.5	206	168
2x-2y	25	909	0.7	110	94	1040	0.7	138	107	842	0.7	100	88	957	0.7	124	101
	40	630	0.9	36	31	671	0.9	46	35	576	0.9	33	29	609	0.9	41	33
#3 Ties		Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy
@12in		150	150	5.40	5.40	150	150	5.40	5.40	135	135	5.40	5.40	135	135	5.40	5.40
.98 X		1180	0.0	350	300	1410	0.0	431	341	1100	0.0	320	285	1300	0.0	389	321
Ar(in^2)	11	1120	0.2	315	270	1330	0.2	388	306	1050	0.2	288	256	1230	0.2	350	289
=3.16		1100	0.3	275	236	1300	0.3	339	268	1030	0.3	252	224	1200	0.3	306	253
	17	1050	0.4	236	203	1230	0.4	291	230	980	0.4	216	192	1140	0.4	263	217
4-# 8	21	992	0.5	196	169	1150	0.5	242	191	923	0.5	180	160	1060	0.5	219	180
2x-2y	25	924	0.7	118	101	1060	0.7	145	115	857	0.7	108	96	970	0.7	131	108
	40	635	0.9	39	33	674	0.9	48	38	581	0.9	36	32	612	0.9	43	36
#3 Ties		Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy
@12in		150	150	5.40	5.40	150	150	5.40	5.40	135	135	5.40	5.40	135	135	5.40	5.40
1.95 X	0	1280	0.0	443	344	1520	0.0	524	384	1210	0.0	413	329	1410	0.0	483	365
Ar(in^2)	11	1220	0.2	399	310	1430	0.2	472	346	1150	0.2	372	296	1330	0.2	434	328
- 6.32	13	1200	0.3	349	271	1400	0.3	413	303	1120	0.3	325	259	1290	0.3	380	287
	17	1140	0.4	299	232	1320	0.4	354	259	1060	0.4	279	222	1220	0.4	326	246
8-# 8	21	1070	0.5	249	193	1220	0.5	295	216	995	0.5	232	185	1130	0.5	271	205
4x-2y	25	987	0.7	149	116	1110	0.7	177	129	918	0.7	139	111	1020	0.7	163	123
	40	654	0.9	49	38	684	0.9	59	43	598	0.9	46	37	621	0.9	54	41
#3 Ties		Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy
@12in		150	150	5.40	5.40	150	150	5.40	5.40	135	135	5.40	5.40	135	135	5.40	5.40
2.93 X	0	1390	0.0	502	437	1630	0.0	583	477	1320	0.0	472	421	1520	0.0	541	457
Ar(in^2)	11	1320	0.2	451	393	1520	0.2	524	429	1240	0.2	424	379	1420	0.2	487	412
=9.48	13	1290	0.3	395	344	1490	0.3	459	375	1210	0.3	371	332	1380	0.3	426	360
	17	1220	0.4	338	295	1390	0.4	393	322	1150	0.4	318	284	1300	0.4	365	309
12-# 8	21	1140	0.5	282	245	1290	0.5	328	268	1070	0.5	265	237	1190	0.5	304	257
4x-4y	25	1050	0.7	169	147	1160	0.7	196	161	975	0.7	159	142	1080	0.7	182	154
	40	669	0.9	56	49	691	0.9	65	53	610	0.9	53	47	626	0.9	60	51
#3 Ties		Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy
@12in		149	149	5.40	5.40	149	149	5.40	5.40	135	135	5.40	5.40	135	135	5.40	5.40
3.85 X	0	1490	0.0	607	458	1730	0.0	688	498	1420	0.0	577	443	1620	0.0	646	479
Ar(in^2)	11	1410	0.2	546	413	1610	0.2	619	448	1330	0.2	519	399	1510	0.2	582	431
=12.48	13	1380	0.3	478	361	1570	0.3	542	392	1300	0.3	454	349	1470	0.3	509	377
	17	1300	0.4	409	309	1470	0.4	464	336	1220	0.4	389	299	1370	0.4	436	323
8-#11	21	1200	0.5	341	258	1350	0.5	387	280	1130	0.5	324	249	1250	0.5	363	269
4x-2y	25	1100	0.7	204	154	1210	0.7	232	168	1030	0.7	194	149	1120	0.7	218	161
	40	679	0.9	68	51	694	0.9	77	56	618	0.9	64	49	627	0.9	72	53
#4 Ties		Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy	Cex	Cey	rmx	rmy
@12in		149	149	5.40	5.40	149	149	5.40	5.40	134	134	5.40	5.40	134	134	5.40	5.40

Notes : 1. Cex = Pex$(KxLx)^2$/10000. (kip-ft^2), Cey = Pey$(KyLy)^2$/10000. (kip-ft^2), KL in ft, rmx & rmy in inches.
2. Zeroes in columns for ϕcPn, Mux, and Muy indicate that no suitable reinforcing bar arrangement is available for the indicated steel percentage.
3. See Figure 2 for definition of bar arrangement (nx-my). NW = Normal wt. concrete.
4. Mux $\phi bMnx$ and Muy $\phi bMny$ when $Pu/(\phi cPn)$ * 0.0

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