Revision and Errata List, March 1, 2003

AISC Design Guide 2: Steel and Composite Beams with Web Openings

The following editorial corrections have been made in the Second Printing, September 1991. To facilitate the incorporation of these corrections, this booklet has been constructed using copies of the revised pages, with corrections noted. The user may find it convenient in some cases to hand-write a correction; in others, a cut-and-paste approach may be more efficient.



 ϕ = 0.90 for steel beams and 0.85 for composite beams

$$P_r = F_y A_r \le \frac{F_y \iota_w u_o}{2\sqrt{3}}$$

Rev.

3/1/03

 A_r = cross-sectional area of reinforcement above or below the opening.

The reinforcement should be extended beyond the opening by a distance $\ell_1 = a_o/4$ or $A_r\sqrt{3}/(2t_w)$, whichever is greater, on each side of the opening (Figs 3.3 and 3.4). Within each extension, the required strength of the weld is

$$R_{wr} = \phi F_y A_r \tag{3-32}$$

If reinforcing bars are used on only one side of the web, the section should meet the following additional requirements.

$$A_r \le \frac{A_f}{3} \tag{3-33}$$

$$\frac{a_o}{h_o} \le 2.5 \tag{3-34}$$

$$\frac{s_t}{t_w} \text{ or } \frac{s_b}{t_w} \le \frac{140}{\sqrt{F_y}}$$
(3-35)

$$\frac{M_u}{V.d} \le 20 \tag{3-36}$$

in which A_f = area of flange

 M_u and V_u = factored moment and shear at centerline of opening, respectively.

6. Spacing of openings

Openings should be spaced in accordance with the following criteria to avoid interaction between openings.

Rectangular openings:
$$S \ge h_o$$
 (3-37a)

$$S \ge a_o \left(\frac{V_u / \phi V_p}{1 - V_u / \phi \overline{V_p}} \right)$$
 (3-37b)

(3-38a)

Circular openings: $S \ge 1.5 D_o$

$$S \ge D_o\left(\frac{V_u/\phi\overline{V_p}}{1-V_u/\phi\overline{V_p}}\right)$$
 (3-38b)

in which S = clear space between openings.

In addition to the requirements in Eqs. 3-37 and 3-38, openings in composite beams should be spaced so that

$$S \ge a_o$$
 (3-39a)

$$S \ge 2.0 \ d \tag{3-39b}$$

Rev. 3/1/03

c. Additional criteria for composite beams

In addition to the guidelines presented above, composite members should meet the following criteria.

1. Slab reinforcement

Transverse and longitudinal slab reinforcement ratios should be a minimum of 0.0025, based on the *gross* area of the slab, within a distance d or a_o , whichever is greater, of the opening. For beams with longitudinal ribs, the transverse reinforcement should be below the heads of the shear connectors.

2. Shear connectors

In addition to the shear connectors used between the high moment end of the opening and the support, a minimum of two studs per foot should be used for a distance d or a_o , whichever is greater, from the high moment end of the opening toward the direction of *increasing* moment.

3. Construction loads

If a composite beam is to be constructed without shoring, the section at the web opening should be checked for adequate strength as a *non-composite* member under factored dead and construction loads.

3.8 ALLOWABLE STRESS DESIGN

The safe and accurate design of members with web openings requires that an ultimate strength approach be used. To accommodate members designed using ASD, the expressions presented in this chapter should be used with $\phi = 1.00$ and a load factor of 1.7 for both dead and live loads. These factors are in accord with the Plastic Design Provisions of the AISC ASD Specification (1978).

Select reinforcement:

Check to see if reinforcement may be placed on one side of web (Eqs. 3-33 through 3-36):

$$A_r \leq \frac{A_f}{3} \qquad ? \qquad \frac{a_o}{h_o} \leq 2.5 ?$$

$$0.652 \leq \frac{7.53 \times 0.630}{3} ? \qquad 1.82 \leq 2.5 \text{ OK}$$

$$0.652 \leq 1.58 \text{ OK}$$

$$\frac{s}{t_w} \leq \frac{140}{\sqrt{F_y}} \qquad ? \qquad \frac{M_u}{V_u d} \leq 20 ?$$

$$\frac{3.555}{0.39} \leq \frac{140}{\sqrt{50}} \qquad ? \qquad \frac{3600}{30 \times 18.11} \leq 20 ?$$

$$91 \leq 198 \text{ OK} \qquad 662 \leq 20 \text{ OK}$$

Therefore, reinforcement may be placed on one side of the web.

From the stability check [Eq. (3-22)], $b/t \le 9.2$. Use $\frac{3}{8} \times 1\frac{3}{4}$ in. bar -b/t = 4.67

 $A_r = 0.656 > 0.65$ assumed OK $d_r = 3.555 - \frac{3}{16} = 3.368$ as assumed

Comer radii (section 3.7b2) and weld design:

The corner radii must be $2t_w = 0.78$ in. $\ge \frac{5}{8}$ in. Use $\frac{7}{8}$ in. or larger.

The weld must develop $R_{wr} = \phi 2P_r = 0.90 \times 2 \times 32.8 =$ 59.0 kips within the length of the opening and $R_{wr} =$



Fig. 4.6. Moment-shear interaction diagram for Example 2.

 $\phi F_y A_r = 0.90 \times 50 \times 0.656 = 29.5$ kips within each extension. Use extensions of $\ell_1 \ge a_o/4 = 20/4 = 5$ in., $\ge \sqrt{3}A_r/(2t_w) = \sqrt{3} \times 0.656/(2 \times 0.39) = 1.46$ in. Use 5 in. The total length of the reinforcement = $20.0 + 2 \times 5.0 = 30.0$ in.

Assume E70XX electrodes, which provide a shear strength of the weld metal $F_w = 0.60 \times 70 = 42$ ksi (AISC 1986a). A fillet weld will be used on one side of the reinforcement bar, within the length of the opening. Each y_{16} in. weld will provide a shear capacity of $\phi F_w a_o \times 0.707 \times y_{16} = 0.75 \times 42 \times 20 \times 0.707 \times y_{16} = 27.8$ kips.

For $R_{wr} = 59.0$ kips, with the reinforcement on one side of the web, 59.0/27.8 = 2.12 sixteenths are required. Use a $\frac{3}{16}$ in. fillet weld. [Note the minimum size of fillet weld for this material is $\frac{3}{16}$ in.]. Welds should be used on both sides of the bar in the extensions. By inspection, the weld size is identical.

According to AISC (1986b), the shear rupture strength of the base metal must also be checked. The shear rupture strength = $\phi F_{wr}A_{ns}$, in which $\phi = 0.75$, $F_n = 0.6F_u$, $F_u =$ tensile strength of base metal, and A_{ns} = net area subject to shear. This requirement is effectively covered for the steel section by the limitation that $P_r \leq F_y t_w a_o/(2\sqrt{3})$ which is based on $\phi = 0.90$ instead of $\phi = 0.75$, but uses $F_y/\sqrt{3} =$ $0.58F_y$ in place of $0.6F_u$. For the reinforcement, the shear rupture force = $\phi 2P_r(1\frac{3}{4} - \frac{3}{16})/1\frac{3}{4} = 52.7$ kips. $\phi F_n A_{ns} =$ $0.75 \times 0.6 \times 58$ ksi x 3/8 in. x 120 in.] = 196 kips ≥ 52.7 , OK.

The completed design is illustrated in Fig. 4.7.

4.5 EXAMPLE 3: COMPOSITE BEAM WITH UNREINFORCED OPENING

Simply supported composite beams form the floor system of an office building. The 36-ft beams are spaced 8 ft apart and support uniform loads of $w_d = 0.608$ kips/ft and $w_l =$ 0.800 kips/ft. The slab has a total thickness of 4 in. and will be placed on metal decking. The decking has 2 in. ribs on 12 in. centers transverse to the steel beam. An A36 W21×44 steel section and normal weight concrete will be used. Normal weight concrete (w = 145 lbs/ft³) with $f'_c = 3$ ksi will be used.

Can an unreinforced 11×22 in. opening be placed at the quarter point of the span? See Fig. 4.8.

Loading:

$$w_{\mu} = 1.2 \times 0.608 + 1.6 \times 0.800 = 2.01$$
 kips/ft

At the quarter point:

$$V_u = \frac{w_u L}{4} = \frac{2.01 \times 36}{4} = 18.1$$
 kips

Chapter 5 BACKGROUND AND COMMENTARY

5.1 GENERAL

This chapter provides the background and commentary for the design procedures presented in Chapter 3. Sections 5.2a through 5.2g summarize the behavior of steel and composite beams with web openings, including the effects of openings on stress distributions, modes of failure, and the general response of members to loading. Section 5.2h provides the commentary for section 3.2 on load and resistance factors, while sections 5.3 through 5.7 provide the commentary for sections 3.3 through 3.7 on design equations and guidelines for proportioning and detailing beams with web openings.

5.2 BEHAVIOR OF MEMBERS WITH WEB OPENINGS

a. Forces acting at opening

The forces that act at opening are shown in Fig. 5.1. In the figure, a composite beam is illustrated, but the equations that follow pertain equally well to steel members. For positive bending, the section below the opening, or bottom tee, is subjected to a tensile force, P_b , shear, V_b , and secondary bending moments, M_{bl} and M_{bh} . The section above the opening, or top tee, is subjected to a compressive force, P_t , shear, V_t , and secondary bending moments, bending moments, M_{tl} and M_{th} . Based on equilibrium,



Fig. 5.1. Forces acting at web opening.

 $P_b = P_t = P \tag{5-1}$

$$V = V_b + V_t \tag{5-2}$$

$$V_b a_o = M_{bl} + M_{bh} \tag{5-3}$$

$$V_t a_o = M_{tl} + M_{th} \tag{5-4}$$

$$M = Pz + M_{th} + M_{bh} - \frac{Va_o}{2}$$
(5-5)

in which

- V = total shear acting at an opening
- M = primary moment acting at opening center line
- $a_o = \text{length of opening}$
- z = distance between points about which secondary bending moments are calculated

b. Deformation and failure modes

The deformation and failure modes for beams with web openings are illustrated in Fig. 5.2. Figures 5.2(a) and 5.2(b) illustrate steel beams, while Figs. 5.2(c) and 5.2(d) illustrate composite beams with solid slabs.

High moment-shear ratio

The behavior at an opening depends on the ratio of moment to shear, *M/V* (Bower 1968, Cho 1982, Clawson & Darwin 1980, Clawson & Darwin 1982a, Congdon & Redwood 1970, Donahey & Darwin 1986, Donahey & Darwin 1988, Granada 1968).

Rev. 3/1/03



Fig. 5.2. Failure modes at web openings, (a) Steel beam, pure bending, (b) steel beam, low moment-shear ratio, (c) composite beam with solid slab, pure bending, (d) composite beam with solid slab, low moment-shear ratio.

Medium and low moment-shear ratio

As M/V decreases, shear and the secondary bending moments increase, causing increasing differential, or Vierendeel, deformation to occur through the opening [Figs. 5.2(b) and 5.2(d)]. The top and bottom tees exhibit a well-defined change in curvature.

For steel beams [Fig. 5.2(b)], failure occurs with the formation of plastic hinges at all four corners of the opening. Yielding first occurs within the webs of the tees.

For composite beams [Fig. 5.2(d)], the formation of the plastic hinges is accompanied by a diagonal tension failure within the concrete due to prying action across the opening. For members with ribbed slabs, the diagonal tension failure is manifested as a rib separation and a failure of the concrete around the shear connectors (Fig. 5.3). For composite members with ribbed slabs in which the rib is parallel to the beam, failure is accompanied by longitudinal shear failure in the slab (Fig. 5.4).

For members with low moment-shear ratios, the effect of secondary bending can be quite striking, as illustrated by the stress diagrams for a steel member in Fig. 5.5 (Bower 1968) and the strain diagrams for a composite member with a ribbed slab in Fig. 5.6 (Donahey & Darwin 1986). Secondary bending can cause portions of the bottom tee to go into compression and portions of the top tee to go into tension, even though the opening is subjected to a positive bending moment. In composite beams, large slips take place between the concrete deck and the steel section over the opening (Fig. 5.6). The slip is enough to place the lower portion of the slab in compression



Fig. 5.3. Rib failure and failure of concrete around shear connectors in slab with transverse ribs.

at the low moment end of the opening, although the adjacent steel section is in tension. Secondary bending also results in tensile stress in the top of the concrete slab at the low moment end of the opening, which results in transverse cracking.

Failure

Web openings cause stress concentrations at the corners of the openings. For steel beams, depending on the proportions of the top and bottom tees and the proportions of the opening with respect to the member, failure can be manifested by general yielding at the corners of the opening, followed by web tearing at the high moment end of the bottom tee and the low moment end of the top tee (Bower 1968, Congdon & Redwood 1970, Redwood & McCutcheon 1968). Strength may be reduced or governed by web buckling in more slender members (Redwood et al. 1978, Redwood & Uenoya 1979). In high moment regions, compression buckling of the top tee is a concern for steel members (Redwood & Shrivastava 1980). Local buckling of the compression flange is not a concern if the member is a compact section (AISC 1986b).

For composite beams, stresses remain low in the concrete until well after the steel has begun to yield (Clawson & Darwin 1982a, Donahey & Darwin 1988). The concrete contributes significantly to the shear strength, as well as the flexural strength of these beams at web openings. This contrasts with the standard design practice for composite beams, in which the concrete deck is used only to resist the bending moment, and shear is assigned solely to the web of the steel section.

For both steel and composite sections, failure at web openings is quite ductile. For steel sections, failure is preceded by large deformations through the opening and significant yielding of the steel. For composite members, failure is preceded by major cracking in the slab, yielding of the steel, and large deflections in the member.

First yielding in the steel does not give a good representation of the strength of either steel or composite sections. Tests show that the load at first yield can vary from 35 to 64 percent of the failure load in steel members (Bower 1968, Congdon & Redwood 1970) and from 17 to 52 percent of the failure load in composite members (Clawson & Darwin <u>1982a</u>, Donahey & Darwin 1988).

Rev. 3/1/03



Fig. 5.4. Longitudinal rib shear failure.

c. Shear connectors and bridging

For composite members, shear connectors above the opening and between the opening and the support strongly affect the capacity of the section. As the capacity of the shear connectors increases, the strength at the opening increases. This increased capacity can be obtained by either increasing the number of shear connectors or by increasing the capacity of the individual connectors (Donahey & Darwin 1986, Donahey & Darwin 1988). Composite sections are also subject to bridging, the separation of the slab from the steel section. Bridging occurs primarily in beams with transverse ribs and occurs more readily as the slab thickness increases (Donahey & Darwin 1986, Donahey & Darwin 1988).

d. Construction considerations

For composite sections, Redwood and Poumbouras (1983) observed that construction loads as high as 60 percent of member capacity do not affect the strength at web openings. Donahey and Darwin (1986, 1988) observed that cutting openings after the slab has been placed can result in a transverse crack. This crack, however, does not appear to affect the capacity at the opening.

e. Opening shape

Generally speaking, round openings perform better than rectangular openings of similar or somewhat smaller size (Redwood 1969, Redwood & Shrivastava 1980). This improved performance is due to the reduced stress concentrations in the region of the opening and the relatively larger web regions in the tees that are available to carry shear.

f. Multiple openings

If multiple openings are used in a single beam, strength can be reduced if the openings are placed too closely together



Fig. 5.5. Stress diagrams for opening in steel beam—low momentshear ratio (Bower 1968).

(Aglan & Redwood 1974, Dougherty 1981, Redwood 1968a, Redwood 1968b, Redwood & Shrivastava 1980). For steel beams, if the openings are placed in close proximity, (1) a plastic mechanism may form, which involves interaction between the openings, (2) the portion of the member between the openings, or web post, may become unstable, or (3) the web post may yield in shear. For composite beams, the close proximity of web openings in composite beams may also be detrimental due to bridging of the slab from one opening to another.

g. Reinforcement of openings

If the strength of a beam in the vicinity of a web opening is not satisfactory, the capacity of the member can be increased by the addition of reinforcement. As shown in Fig. 5.7, this reinforcement usually takes the form of longitudinal steel bars which are welded above and below the opening (U.S. Steel 1986, Redwood & Shrivastava 1980). To be effective, the bars must extend past the corners of the opening in order to ensure that the yield strength of the bars is fully developed. These bars serve to increase both the primary and secondary flexural capacity of the member.



Fig. 5.6. Strain distributions for opening in composite beam—low moment-shear ratio (Donahey & Darwin 1988).



Fig. 5.7. Reinforced opening.

h. Load and resistance factors

The design of members with web openings is based on strength criteria rather than allowable stresses because the elastic response at web openings does not give an accurate prediction of strength or margin of safety (Bower 1968, Clawson & Darwin 1982, Congdon & Redwood 1970, Donahey & Darwin 1988).

The load factors used by AISC (1986b) are adopted. If alternate load factors are selected for the structure as a whole, they should also be adopted for the regions of members with web openings.

The resistance factors, $\phi = 0.90$ for steel members and $\phi = 0.85$ for composite members, coincide with the values of ϕ used by AISC (1986b) for flexure. The applicability of these values to the strength of members at web openings was established by comparing the strengths predicted by the design expressions in Chapter 3 (modified to account for actual member dimensions and the individual yield strengths of the flanges, webs, and reinforcement) with the strengths of 85 test specimens (Lucas & Darwin 1990): 29 steel beams with unreinforced openings [19] with rectangular openings (Bower 1968, Clawson & Darwin 1980, Congdon & Redwood 1970, Cooper et al. 1977, Redwood et al. 1978, Redwood & McCutcheon 1968) and 10 with circular openings (Redwood et al. 1978, Redwood & McCutcheon 1968)], 21 steel beams with reinforced openings (Congdon & Redwood 1970, Cooper & Snell 1972, Cooper et al. 1977, Lupien & Redwood 1978), 21 composite beams with ribbed slabs and unreinforced openings (Donahey & Darwin 1988, Redwood & Poumbouras 1983, Redwood & Wong 1982), 11 composite beams with solid slabs and unreinforced openings (Cho 1982, Clawson & Darwin 1982, Granade 1968), and 3 composite beams with reinforced openings (Cho 1982, Wiss et al. 1984). Resistance factors of 0.90 and 0.85 are also satisfactory for two other design methods discussed in this chapter (see Eqs. 5-7 and 5-29) (Lucas & Darwin 1990).

5.3 DESIGN OF MEMBERS WITH WEB OPENINGS

The interaction between the moment and shear strengths at an opening are generally quite weak for both steel and composite sections. That is, at openings, beams can carry a large percentage of the maximum moment capacity without a reduction in the shear capacity and vice versa.

The design of web openings has historically consisted of the construction of a moment-shear interaction diagram of the type illustrated in Fig. 5.8. Models have been developed to generate the moment-shear diagrams point by point (Aglan & Qaqish 1982, Clawson & Darwin 1983, Donahey & Darwin 1986, Poumbouras 1983, Todd & Cooper 1980, Wang et al. 1975). However, these models were developed primarily for research. For design it is preferable to generate the interaction diagram more simply. This is done by calculating the maximum moment capacity, M_m , the maximum shear capacity, V_m , and connecting these points with a curve or series of straight line segments. This has resulted in a number of different shapes for the interaction diagrams, as illustrated in Figs. 5.8 and 5.9.

To construct a curve, the end points, M_m and V_m , must be determined for all models. Some other models require, in addition, the calculation of M_v , which represents the maximum moment that can be carried at the maximum shear [Fig. 5.9(a), 5.9(b)].

Virtually all procedures agree on the maximum moment capacity, M_m . This represents the bending strength at an opening subjected to zero shear. The methods differ in how they calculate the maximum shear capacity and what curve shape is used to complete the interaction diagram.

Models which use straight line segments for all or a portion of the curve have an apparent advantage in simplicity of construction. However, models that use a single curve, of the type shown in Fig. 5.9(c), generally prove to be the easiest to apply in practice.

Historically, the maximum shear capacity, V_m , has been calculated for specific cases, such as concentric unreinforced openings (Redwood 1968a), eccentric unreinforced openings (Kussman & Cooper 1976, Redwood 1968a, Redwood & Shrivastava 1980, Wang et al. 1975), and eccentric reinforced openings (Kussman & Cooper 1976, Redwood 1971, Redwood



Fig. 5.8. General moment-shear interaction diagram (Darwin & Donahey 1988).

Rev. 3/1/03 & Shrivastava 1980, Wang et al, 1975) in steel beams; and concentric and eccentric unreinforced openings (Clawson & Darwin 1982a, Clawson & Darwin 1982b, Darwin & Donahey 1988, Redwood & Poumbouras 1984, Redwood & Wong 1982) and reinforced openings (Donoghue 1982) in composite beams. Until recently (Lucas & Darwin 1990), there has been little connection between shear capacity expressions for reinforced and unreinforced openings or for openings in steel and composite beams. The result has been a series of special-



Fig. 5.9. Moment-shear interaction diagrams, (a) Constructed using straight line segments, (b) constructed using multiple junctions (Redwood & Poumbouras 1983), (c) constructed using a single curve (Clawson & Darwin 1980, Darwin & Donahey 1988).

ized equations for each type of construction (U.S. Steel 1986, U.S. Steel 1984, U.S. Steel 1981). As will be demonstrated in section 5.6, however, a single approach can generate a family of equations which may be used to calculate the shear capacity for openings with and without reinforcement in both steel and composite members.

The design expressions for composite beams are limited to positive moment regions because of a total lack of test data for web openings in negative moment regions. The dominant effect of secondary bending in regions of high shear suggests that the concrete slab will contribute to shear strength, even in negative moment regions. However, until test data becomes available, opening design in these regions should follow the procedures for steel beams.

The following sections present design equations to describe the interaction curve, and calculate the maximum moment and shear capacities, M_m and V_m .

5.4 MOMENT-SHEAR INTERACTION

The weak interaction between moment and shear strengths at a web opening has been dealt with in a number of different ways, as illustrated in Figs. 5.8 and 5.9. Darwin and Donahey (1988) observed that this weak interaction can be conveniently represented using a cubic interaction curve to relate the nominal bending and shear capacities, M_n and V_n , with the maximum moment and shear capacities, M_m and V_m (Fig. 5.10).



the expressions to be simplified. For $e \neq 0$, the plastic neutral axis, PNA, will be located within the reinforcing bar at the edge of the opening closest to the centroid of the original steel section.

For members with larger eccentricities [Fig. 5.11(c)], i.e., $e \ge F_{vr}A_r/(F_vt_w)$, the maximum moment capacity is

$$M_m = M_p - F_y \Delta A_s \left(\frac{h_o}{4} + e_{\perp}\right) + F_{yr} \Delta A_s \frac{A_r}{2t_w} \le M_p \quad (5-12a)$$

$$M_m = F_y \left[Z - \Delta A_s \left(\frac{h_o}{4} + e - \frac{F_{yr} A_r}{F_y 2t_w} \right) \right] \le M_p \quad (5-12b)$$

in which $\Delta A_s = h_o t_w - 2A_r F_{yr}/F_y$

Like Eq. 5-11, Eq. 5-12 is based on the assumptions that the reinforcement is concentrated along the top and bottom edges of the opening and that the thickness of the reinforcement is small. In this case, however, the PNA lies in the web of the larger tee. For $A_r = 0$, Eqs. 5-12a and b become identically Eq. 5-10.

In Chapter 3, Eqs. 3-7 and 3-8 are obtained from Eqs. 5-11 and 5-12, respectively, by factoring $M_p = F_y Z$ from the terms on the right-hand side of the equations and making the substitution $F_{yr} = F_y$.

The moment capacity of reinforced openings is limited to the plastic bending capacity of the unperforated section (Redwood & Shrivastava 1980, Lucas and Darwin 1990).

b. Composite beams

Figure 5.12 illustrates stress diagrams for composite sections in pure bending. For a given beam and opening configuration, the force in the concrete, P_c , is limited to the lower of the concrete compressive strength, the shear connector capacity, or the yield strength of the net steel section.

$$P_c \leq 0.85 f'_c b_e t_e \tag{5-13a}$$

$$\leq NQ_n$$
 (5-13b)

$$\leq T' = F_{y}A_{sn} \tag{5-13c}$$

in which A_{sn} = net steel area = $A_s - h_o t_w + 2A_r F_{yr}/F_y$ The maximum moment capacity, M_m , depends on which

of the inequalities in Eq. 5-13 governs. If $P_c = T'$ [Eq. 5-13c and Fig. 5.12(a)],

$$M_m = T'\left(\frac{d}{2} + \frac{\Delta A_s e}{A_{sn}} + t_s - \frac{\bar{a}}{2}\right) \quad (5-14)$$

in which $\Delta A_s = h_o t_w - 2A_r F_{vr}/F_v$

 \bar{a} = depth of concrete compression block = $P_c/(0.85f'_c b_e)$ for solid slabs and ribbed slabs for which $\bar{a} \leq t'_s$.

If $\overline{a} > t'_s$, as it can be for ribbed slabs with longitudinal ribs, the term $\left(t_s - \frac{\overline{a}}{2}\right)$ in Eq. 5-14 must be replaced with the appropriate expression for the distance between the

top of the steel flange and the centroid of the concrete force. If $P_c < T'$ (Eq. 5-13a or 5-13b), a portion of the steel section is in compression. The plastic neutral axis, PNA, may be in either the flange or the web of the top tee, based on the inequality:

$$P_c + F_y A_f \stackrel{<}{\underset{>}{\sim}} F_y (A_{sn} - A_f) \tag{5-15}$$

in which A_f = the flange area = $b_f t_f$.

If the left side of Eq. 5-15 exceeds the right side, the PNA is in the flange [Fig. 5.12b] at a distance $x = (A_{yn}F_y)$ $P_c)/(2b_f F_v)$ from the top of the flange. In this case,

$$M_{m} = T' \left(\frac{d}{2} + \frac{\Delta A_{s}e - b_{f} x^{2}}{A_{sn}} \right) + P_{c} \left(t_{s} - \frac{\bar{a}}{2} \right)$$
(5-16)



Fig. 5.11. Steel sections in pure bending, (a) Unreinforced opening, (b) reinforced opening, $e \leq F_{yr}A_r/(F_y t_w, (c) reinforced opening, e \geq F_{yr}A_r/(F_y t_w).$

The capacity at the opening, V_m , is obtained by summing the individual capacities of the bottom and top tees.

$$V_m = V_{mb} + V_{mt}$$
 (5-18)

 V_{mb} and V_{mt} are calculated using the moment equilibrium equations for the tees, Eq. 5-3 and 5-4, and appropriate representations for the stresses in the steel, and if present, the concrete and opening reinforcement. Since the top and bottom tees are subjected to the combined effects of shear and secondary bending, interaction between shear and axial stresses must be considered in order to obtain an accurate representation of strength. The greatest portion of the shear is carried by the steel web.

The interaction between shear and normal stress results in a reduced axial strength, $\overline{F_y}$, for a given material strength, F_y , and web shear stress, τ , which can be represented using the von Mises yield criterion.

$$\overline{F}_{y} = (F_{y}^{2} - 3\tau^{2})^{\frac{1}{2}}$$
(5-19)

The interaction between shear and axial stress is not considered for the concrete. However, the axial stress in the concrete is assumed to be $0.85f'_c$ when V_m is obtained.

The stress distributions shown in Fig. 5.13, combined with Eqs. 5-3 and 5-4 and Eq. 5-19, yield third order equations in V_{mb} and V_{mt} . These equations must be solved by iteration, since a closed-form solution cannot be obtained (Clawson & Darwin 1980).

For practical design, however, closed-form solutions are desirable. Closed-form solutions require one or more additional simplifying assumptions, which may include a simplified version of the von Mises yield criteria (Eq. 5-19), limiting neutral axis locations in the steel tees to specified locations, or ignoring local equilibrium within the tees.

As demonstrated by Darwin & Donahey (1988), the form of the solution for V_{mb} and V_{mt} depends on the particular assumptions selected. The expressions in Chapter 3 use a sim-



Fig. 5.13. Axial stress distributions for opening at maximum shear.

plified version of the von Mises criterion and ignore some aspects of local equilibrium within the tees. Other solutions may be obtained by using fewer assumptions, such as the simplified version of the von Mises criterion only or ignoring local equilibrium within the tees only. The equations used in Chapter 3 will be derived first, followed by more complex expressions.

a. General equation

A general expression for the maximum shear capacity of a tee is obtained by considering the most complex configuration, that is, the composite beam with a reinforced opening. Expressions for less complex configurations are then obtained by simply removing the terms in the equation corresponding to the concrete and/or the reinforcement.

The von Mises yield criterion, Eq. 5-19, is simplified using a linear approximation.

$$\overline{F}_{y} = \lambda F_{y} - \tau \sqrt{3} \tag{5-20}$$

The term λ can be selected to provide the best fit with data. Darwin and Donahey (1988) used $\lambda = (1 + \sqrt{2})/2 =$ 1.207..., for which Eq. 5-20 becomes the linear best uniform approximation of the von Mises criterion. More recent research (Lucas & Darwin 1990) indicates that $\lambda = \sqrt{2} =$ 1.414... gives a better match between test results and predicted strengths. Figure 5.14 compares the von Mises criterion with Eq. 5-20 for these two values of λ . As illustrated in Fig. 5.14, a maximum shear cutoff, $\tau \leq F_{\nu}/\sqrt{3}$, based on the von Mises criterion, is applied. Figure 5.14 also shows that the axial stress, \overline{F}_{y} , may be greatly overestimated for low values of shear stress, τ . However, the limitations on p_o (section 3.7a2) force at least one tee to be stocky enough (low value of ν) that the calculated value of V_m is conservative. In fact, comparisons with tests of steel beams show that the predicted strengths are most conserva-



Fig. 5.14. Yield functions for combined shear and axial stress.

however, provides a straightforward solution for both steel and composite beams, as illustrated in Examples 2 and 4 in Chapter 3.

Alternate equations for V_{mb} and V_{mt} :

If the full von Mises criterion (Eq. 5-19) is used, instead of the linear approximation (Eq. 5-20), to represent \overline{F}_y in Eq. 5-21, a quadratic equation is obtained for V_{mt} . The solution of that equation takes a somewhat more complex form than Eq. 5-21.

$$V_{mt} = V_{pt} \left[\frac{\mu \nu + (3\nu^2 - 3\mu^2 + 9)^{\frac{1}{2}}}{3 + \nu^2} \right] \le 1$$
 (5-27)

in which V_{pt} , ν and μ are as previously defined. For noncomposite tees without reinforcement, Eq. 5-27 takes a simpler form.

$$V_{mt} = V_{pt} \left(\frac{3}{3+\nu^2}\right)^{\nu_2} \le 1$$
 (5-28)

Equations 5-27 and 5-28 are identical with those used by Redwood and Poumbouras (1984) and by Darwin and Donahey (1988) in their "Solution II." These equations completely satisfy the von Mises criterion, but, perhaps surprisingly, do not provide a closer match with experimental data than Eq. 5-22 (Lucas & Darwin 1990).

To obtain a better match with experimental results requires another approach (Darwin & Donahey 1988, Lucas & Darwin 1990). This approach uses the linear approximation for the von Mises criterion (Eq. 5-19) to control the interaction between shear and normal stresses within the web of the steel tee, but uses a stress distribution based on the full crosssection of the steel tee (Fig. 5.13) to develop the secondary moment equilibrium equation (Eq. 5-4). The PNA is assumed to fall in the flange of the steel tee; its precise location is accounted for in the solution for V_{mt} .

 V_{mt} is expressed as follows:

$$V_{mt} = F_y \left(\frac{\beta_t - \sqrt{\beta_t^2 - 4\alpha_t \gamma_t}}{2\alpha_t} \right)$$
 (5-29)

in which
$$\alpha_t = 3 + 2\sqrt{3} \frac{a_o}{s_t}$$

3/1/03

$$\beta_{t} = 2\sqrt{3}(b_{f} - t_{w})\left(s_{t} - t_{f} + \frac{t_{f}^{2}}{s_{t}}\right) + 2\sqrt{3}\lambda t_{w}s_{t} + 2a_{o}[b_{f} + (\lambda - 1)t_{w}] + \frac{2\sqrt{3}}{2}(2Pd_{o} + P_{o}d_{o} - P_{o}d_{o}) + \frac{\sqrt{3}}{2}(P_{o} - P_{o} - P_{o}) + \frac{\sqrt{3}}{2}(P_{o} -$$

$$+ \frac{1}{s_t F_y} (2P_r d_r + P_{ch} d_h - P_{cl} d_{d}) + \frac{1}{F_y} (P_{ch} - P_{cl} - 2P_r)$$

$$\gamma_t = (b_f - t_w)^2 t_f^2 + \lambda^2 t_w^2 s_t^2 + 2\lambda t_w (b_f - t_w) (s_t^2 - s_t t_f + t_f^2)$$

$$+ \frac{2[b_{f} + (\lambda - 1)t_{w}]}{F_{y}}(2P_{r}d_{r} + P_{ch}d_{h} - P_{cl}d_{l}) - ||_{3/1/03}$$

$$\frac{(2P_{r}^{2} + P_{ch}^{2} + P_{cl}^{2})}{2F_{y}^{2}}$$

$$+ \frac{[(b_{f} - t_{w})t_{f} + \lambda t_{w}s_{l}]}{F_{y}}(P_{ch} - P_{cl} - 2P_{r}) + \frac{P_{r}(P_{ch} - P_{cl})}{F_{y}^{2}}$$

Equation 5-29 is clearly more complex than Eqs. 5-22 and 5-27 and is best suited for use with a programmable calculator or computer. It has the advantages that it accounts for the actual steel section and does not require a separate calculation for \bar{s} when reinforcement is used. With $\lambda = \sqrt{2}$, Eq. 5-29 produces a closer match with the experimental data than the other two options (Lucas & Darwin 1990). However, since the flange is included in the calculations, Eq. 5-29 cannot be used to produce a general design aid.

Expressions for tees without concrete and/or opening reinforcement can be obtained from Eqs. 5-29 by setting P_{ch} and P_{cl} or P_r to zero, as required.

b. Composite beams

As explained in Chapter 3, a number of additional expressions are required to calculate the shear capacity of the top tee in composite beams.

The forces in the concrete at the high and low moment ends of the opening, P_{ch} and P_{cl} , and the distances to these forces from the top of the flange of the steel section, d_h and d_h are calculated using Eqs. 3-15a through 3-18b. P_{ch} is limited by the force in the concrete, based on an average stress of $0.85f'_c$, $0.85f'_c b_e t_e$, the stud capacity between the high moment end of the opening and the support, NQ_n , and the tensile capacity of the top tee steel section, $F_v A_{st}$. The third limitation $(F_v A_{st})$ was not originally used in conjunction with Eqs. 5-22 and 5-27, because it was felt to be inconsistent with a model (Fig. 5-15) that ignored the flange of the steel tee (Darwin & Donahey 1988, Donahey & Darwin 1986). Lucas and Darwin (1990), however, have shown that generally improved solutions are obtained when all these limitations are used in conjunction with Eqs. 5-22 and 5-27, as well as Eq. 5-29 which considers the flange.

The number of studs, N, used for the calculation of P_{ch} includes the studs between the high moment end of the opening and the support, not the point of zero moment. This change from normal practice takes into account the large amount of slip that occurs between the slab and the steel section at openings, which tends to mobilize stud capacity, even studs in negative moment regions (Darwin & Donahey 1988, Donahey & Darwin 1986, Donahey & Darwin 1988). To use the more conservative approach will greatly underestimate the shear capacity of openings placed at a point of contraflexure (Donahey & Darwin 1986).

Chapter 6 DEFLECTIONS

6.1 GENERAL

A web opening may have a significant effect on the deflections of a beam. In most cases, however, the influence of a single web opening is small.

The added deflection caused by a web opening depends on its size, shape, and location. Circular openings have less effect on deflection than rectangular openings. The larger the opening and the closer the opening is to a support, the greater will be the increase in deflection caused by the opening. The greatest deflection through the opening itself will occur when the opening is located in a region of high shear. Rectangular openings with a depth, h_o , up to 50 percent of the beam depth, d, and circular openings with a diameter, D_o , up to 60 percent of d_o , cause very little additional deflection (Donahey 1987, Redwood 1983). Multiple openings can produce a pronounced increase in deflection.

As a general rule, the increase in deflection caused by a single large rectangular web opening is of the same order of magnitude as the deflection caused by shear in the same beam without an opening. Like shear deflection, the shorter the beam, the greater the deflection caused by the opening relative to the deflection caused by flexure.

6.2 DESIGN APPROACHES

Web openings increase deflection by lowering the moment of inertia at the opening, eliminating strain compatibility between the material in the top and bottom tees, and reducing the total amount of material available to transfer shear (Donahey 1987, Donahey & Darwin 1986). The reduction in gross moment of inertia increases the curvature at openings, while the elimination of strain capability and reduction in material to transfer shear increase the differential, or Vierendeel, deflection across the opening. The Vierendeel deformation is usually of greater concern than is the local increase in curvature.

A number of procedures have been developed to calculate deflections for flexural members with web openings. Three procedures specifically address steel beams (Dougherty 1980, McCormick 1972a, ASCE 1973), and one method covers composite members (Donahey 1987, Donahey & Darwin 1986). The first three procedures calculate deflections due to the web opening that are added to the deflection of the beam without an opening. The method developed for composite members, which can also be used for steel beams, calculates total deflections of members with web openings. Three of these methods will now be briefly described.

6.3 APPROXIMATE PROCEDURE

The Subcommittee on Beams with Web Openings of the Task Committee on Flexural Members of the Structural Division of ASCE (1971) developed an approximate procedure that represents the portion of the beam from the low moment end of the opening to the far end of the beam as a hinged, propped cantilever (Fig. 6.1). The method was developed for beams with concentric openings. The shear at the opening, V, is evenly distributed between the top and bottom tees. The deflection through the opening, Δ_o , is

$$\Delta_o = \frac{V a_o^3}{6E I_T} \tag{6-1}$$

Rev

3/1/03

Rev. 3/1/03

in which

 a_o = length of opening

E =modulus of elasticity of steel

 I_T = moment of inertia of tee

The additional deflection, Δ_{p_1} , at any point between the high moment end of the opening and the support caused by the opening (Fig. 6.1) is expressed as

$$\Delta_{p_1} = \Delta_o \frac{l_2}{l_o} \tag{6-2}$$

in which

 l_o = distance from high moment end of opening to adjacent support (Fig. 6.1)



Fig. 6.1. Deflections due to web opening—approximate approach (ASCE 1971).

 l_2 = distance from support to point at which deflection is calculated (Fig 6.1)

To enforce slope continuity at the high moment end of the opening, an additional component of deflection, Δ_{p_2} , is obtained.

$$\Delta_{p_2} = \frac{2(\theta_H + \theta_T)l_2l_3}{l_2 + l_2}$$
(6-3)

in which

 $\theta_{H} = \frac{\Delta_{o}}{l_{o}}$ $\theta_{T} = \frac{Va_{o}^{\Box}}{4EI_{T}}$ $l_{3} = l_{o} - l_{2}$

The sum of the displacements calculated in Eqs. 6-2 and 6-3, $\Delta_{p_1} + \Delta_{p_2}$, is added to the deflection obtained for the beam without an opening. The procedure does not consider the deflection from the low moment end of the opening to the adjacent support, slope compatibility at the low moment end of the opening, axial deformation of the tees, or shear deformation in the beam or through the opening. The subcommittee reported that the procedure is conservative.

McCormick (1972b) pointed out that the subcommittee procedure is conservative because of a lack of consideration of compatibility between the axial deformation of the tees and the rest of the beam. He proposed an alternate procedure in which points of contraflexure are assumed at the center line of the opening (McCormick 1972a). Bending and shear deformation of the tees are included but compatibility at the ends of an opening is not enforced. McCormick made no comparison with experimental results.

6.4 IMPROVED PROCEDURE

Dougherty (1980) developed a method in which the deflection due to Vierendeel action at a web opening is obtained (Fig. 6.2). The calculations take into account deformations due to both secondary bending and shear in the tee sections above and below the opening and slope compatibility at the ends of the opening. The increased curvature under primary bending due to the locally reduced moment of inertia at the opening is not included. Shear is assigned to the tees in proportion to their relative stiffnesses, which take into account both flexural and shear deformation.

As shown in Fig. 6.2, Δ_o , θ_1 , θ_2 fully define the deflection throughout a beam due to deflection through the opening. The total deflection through a concentric opening Δ_o is

$$\Delta_o = \Delta_{ob} + \Delta_{os} + \frac{L}{2} (\theta_1 - \theta_2) \qquad (6-4)$$

in which

$$\Delta_{ob} = \frac{Va_o^3}{24EI_T} \tag{6-5}$$

$$\Delta_{os} = \frac{KVa_o}{2GA_T} \tag{6-6}$$

$$\theta_{1} = \frac{Va_{o}^{2}[2I_{T}(2l_{o} + a_{o}) + I_{o}(2l_{o} - a_{o})]}{48EI_{o}I_{T}L} + \frac{\Delta_{os}(2l_{o} + a_{o})}{2Ll_{o}}$$
(6-7)

$$\theta_{2} = \frac{Va_{o}^{2}[2I_{T}(2l_{I} + a_{o}) + I_{o}(2l_{I} + 3a_{o})]}{48EI_{o}I_{T}L} + \frac{\Delta_{os}(2l_{I} + a_{o})}{2Ll_{o}}$$
(6-8)

- V = shear at opening center line
- $G = \text{shear modulus} = E/2(1 + \nu)$
- ν = Poisson's ratio K = shape factor (Knostman et al. 1977)
- K = shape factor (Knostman A_T = area of tee
- I_o = moment of inertia of perforated beam
- L =length of beam
- l_o = distance from high moment end of opening to adjacent support (Fig. 6.2)
- l_1 = distance from low moment end of opening to adjacent support (Fig. 6.2)

The reader is referred to Dogherty (1980) for the case of eccentric openings.

The procedure can, in principle, be used to calculate deflection due to an opening in a composite beam as well as a steel beam. In that case, based on the work of Donahey and Darwin (1986, 1987) described in the next section, the moment of inertia of the top tee should be based on the steel tee only, but I_o should be based on the composite section at the opening.



Rev

3/1/03

Rev. 3/1/03



Fig. 6.2. Deflections due to web opening—improved procedure (Dougherty 1980).

Rev. 3/1/03

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Rev.

3/1/03

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ite Rev. 3/1/03

INDEX

bearing stiffeners, 15 behavior, 37 bottom tee, 3 bridging, 3, 39 bridging in, 50 Rev. 3/1/03 circular openings, 15, 16, 21, 49, 51 compact section, 38 compact sections, 42 composite beam, 11, 43, 47 deflections, 51 deformation, 37 design interaction curves, 8 detailing, 21 detailing beams, 12, 48 dimensions, 49 failure, 38 failure modes, 37 general yielding, 38 high moment end, 3 Rev. 3/1/03 interaction curves, 8, 42, 59 lateral bracing, 49 lateral buckling, 14, 49 local buckling, 13, 38, 48 low moment end, 3 matrix analysis, 53 moment-shear interaction, 8 multiple openings, 39, 51 opening, 49 opening configurations, 9 opening dimensions, 15, 21, 25, 32 opening parameter, 3, 13, 48

opening shape, 39 plastic hinges, 38 plastic neutral axis, 3 post-crack strength, 50 primary bending moment, 3 proportioning, 12, 21, 48 rectangular openings, 16 reinforced opening, 24 reinforced openings, 9, 30, 42 reinforced web openings, 15, 18, 20 reinforcement, 3, 15, 21, 27, 33, 35, 39, 50 reinforcement, slab, 3 resistance factors, 7 secondary bending, 38 secondary bending moments, 3, 44 shear capacity, 10 shear connectors, 16, 21, 39, 50 slab reinforcement, 16, 21, 35, 50 spacing of openings, 16, 21 stability, 21, 35, 48 stability considerations, 12 tee, 3 top tee, 3 unperforated member, 3 unreinforced, 30 unreinforced opening, 22, 27 unreinforced openings, 9, 42 unreinforced web openings, 15, 17, 19 Vierendeel, 38, 51, 52 von Mises, 45 web buckling, 13, 48

Rev. 3/1/03