
Load and Resistance Factor Design Specification for Single-Angle Members

November 10, 2000

Supersedes the *Specification for Load and Resistance
Factor Design of Single-Angle Members* dated
December 1, 1993

Prepared by the
American Institute of Steel Construction, Inc.
Under the Direction of the
AISC Committee on Specifications and approved by
the AISC Board of Directors



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PREFACE

The intention of the AISC Specification is to cover the common everyday design criteria in routine design office usage. It is not feasible to also cover the many special and unique problems encountered within the full range of structural design practice. This separate Specification and Commentary addresses one such topic—single-angle members—to provide needed design guidance for this more complex structural shape under various load and support conditions.

The revised single-angle design criteria were developed through a consensus process by the AISC Task Committee 12 on Single Angles:

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The full AISC Committee on Specifications has reviewed and approved this Specification.

A non-mandatory Commentary provides background for the Specification provisions and the user is encouraged to consult it.

The principal changes in this edition include:

Revisions to flexural design strength criteria

- a. For the limit state of local buckling when the angle leg is in compression
- b. For the limit state of yielding when the tip of an angle leg is in tension
- c. For the limit state of lateral-torsional buckling
- d. For bending about geometric axes

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Load and Resistance Factor Design Specification for Single-Angle Members

November 10, 2000

1. SCOPE

This document contains Load and Resistance Factor Design (LRFD) criteria for hot-rolled, single-angle members with equal and unequal legs in tension, shear, compression, flexure, and for combined forces. It is intended to be compatible with, and a supplement to, the 1999 AISC *Load and Resistance Factor Design Specification for Structural Steel Buildings* and repeats some common criteria for ease of reference. For design purposes, the conservative simplifications and approximations in the Specification provisions for single angles are permitted to be refined through a more precise analysis. As an alternative to this Specification, the 1989 AISC *Specification for Allowable Stress Design of Single-Angle Members* is permitted.

The Specification for single-angle design supersedes any comparable but more general requirements of the AISC LRFD. All other design, fabrication, and erection provisions not directly covered by this document shall be in compliance with the AISC LRFD. For design of slender, cold-formed steel angles, the AISI *LRFD Specification for the Design of Cold-Formed Steel Structural Members* referenced in Section A6 of the AISC LRFD is applicable.

2. TENSION

The tensile design strength $\phi_t P_n$ shall be the lower value obtained according to the limit states of yielding, $\phi_t = 0.9$, $P_n = F_y A_g$, and fracture, $\phi_t = 0.75$, $P_n = F_u A_e$.

- a. For members connected by bolting, the net area and effective net area shall be determined from AISC LRFD Specification Sections B1 to B3 inclusive.
- b. When the load is transmitted by longitudinal welds only or a combination of longitudinal and transverse welds through just one leg of the angle, the effective net area A_e shall be:

$$A_e = A_g U \quad (2-1)$$

where

$$\begin{aligned}
 A_g &= \text{gross area of member} \\
 U &= \left(1 - \frac{\bar{x}}{l}\right) \leq 0.9 \\
 \bar{x} &= \text{connection eccentricity} \\
 l &= \text{length of connection in the direction of loading}
 \end{aligned}$$

- c. When a load is transmitted by transverse weld through just one leg of the angle, A_e is the area of the connected leg and $U = 1$.

For members whose design is based on tension, the slenderness ratio l/r preferably should not exceed 300. Members in which the design is dictated by tension loading, but which may be subject to some compression under other load conditions, need not satisfy the compression slenderness limits.

3. SHEAR

For the limit state of yielding in shear, the shear stress, f_{uv} , due to flexure and torsion shall not exceed:

$$\begin{aligned}
 f_{uv} &\leq \phi_v 0.6 F_y \\
 \phi_v &= 0.9
 \end{aligned} \tag{3-1}$$

4. COMPRESSION

The design strength of compression members shall be $\phi_c P_n$

where

$$\begin{aligned}
 \phi_c &= 0.90 \\
 P_n &= A_g F_{cr}
 \end{aligned}$$

- a. For $\lambda_c \sqrt{Q} \leq 1.5$

$$F_{cr} = Q(0.658^{2\lambda_c^2})F_y \tag{4-1}$$

- b. For $\lambda_c \sqrt{Q} > 1.5$

$$F_{cr} = \left[\frac{0.877}{\lambda_c^2} \right] F_y \tag{4-2}$$

where

$$\lambda_c = \frac{Kl}{r\pi} \sqrt{\frac{F_y}{E}}$$

F_y = specified minimum yield stress of steel

Q = reduction factor for local buckling

The reduction factor Q shall be:

$$\text{when } \frac{b}{t} \leq 0.446 \sqrt{\frac{E}{F_y}}:$$

$$Q = 1.0 \quad (4-3a)$$

$$\text{when } 0.446 \sqrt{\frac{E}{F_y}} < \frac{b}{t} < 0.910 \sqrt{\frac{E}{F_y}}:$$

$$Q = 1.34 - 0.761 \frac{b}{t} \sqrt{\frac{F_y}{E}} \quad (4-3b)$$

$$\text{when } \frac{b}{t} \geq 0.910 \sqrt{\frac{E}{F_y}}:$$

$$Q = \frac{0.534E}{F_y \left(\frac{b}{t}\right)^2} \quad (4-3c)$$

where

b = full width of longest angle leg
 t = thickness of angle

For members whose design is based on compressive force, the largest effective slenderness ratio preferably should not exceed 200.

5. FLEXURE

The flexure design strengths of Section 5.1 shall be used as indicated in Sections 5.2 and 5.3.

5.1. Flexural Design Strength

The flexural design strength shall be limited to the minimum value $\phi_b M_n$ determined from Sections 5.1.1, 5.1.2, and 5.1.3, as applicable, with $\phi_b = 0.9$.

5.1.1. For the limit state of local buckling when the tip of an angle leg is in compression:

$$\text{when } \frac{b}{t} \leq 0.54 \sqrt{\frac{E}{F_y}}:$$

$$M_n = 1.5 F_y S_c \quad (5-1a)$$

when $0.54 \sqrt{\frac{E}{F_y}} < \frac{b}{t} \leq 0.91 \sqrt{\frac{E}{F_y}}$:

$$M_n = F_y S_c \left[1.5 - 0.93 \left(\frac{b/t}{0.54 \sqrt{\frac{E}{F_y}}} - 1 \right) \right] \quad (5-1b)$$

when $\frac{b}{t} > 0.91 \sqrt{\frac{E}{F_y}}$:

$$M_n = 1.34 Q F_y S_c \quad (5-1c)$$

where

b = full width of angle leg with tip in compression

Q = reduction factor per Equation 4-3c

S_c = elastic section modulus to the tip in compression relative to axis of bending

E = modulus of elasticity

5.1.2. For the limit state of yielding when the tip of an angle leg is in tension

$$M_n = 1.5 M_y \quad (5-2)$$

where

M_y = yield moment about the axis of bending

5.1.3. For the limit state of lateral-torsional buckling:

when $M_{ob} \leq M_y$:

$$M_n = [0.92 - 0.17 M_{ob}/M_y] M_{ob} \quad (5-3a)$$

when $M_{ob} > M_y$:

$$M_n = [1.92 - 1.17 \sqrt{M_y/M_{ob}}] M_y \leq 1.5 M_y \quad (5-3b)$$

where

M_{ob} = elastic lateral-torsional buckling moment, from Section 5.2 or 5.3 as applicable

5.2. Bending about Geometric Axes

5.2.1. a. Angle bending members with lateral-torsional restraint along the length shall be designed on the basis of geometric axis bending with the nominal flexural strength M_n limited to the provisions of Sections 5.1.1 and 5.1.2.

- b. For equal-leg angles if the lateral-torsional restraint is only at the point of maximum moment, the required moment shall be limited to $\phi_b M_n$ per Section 5.1. M_y shall be computed using the geometric axis section modulus and M_{ob} shall be substituted by using 1.25 times M_{ob} computed from Equation 5-4.

5.2.2. Equal-leg angle members without lateral-torsional restraint subjected to flexure applied about one of the geometric axes are permitted to be designed considering only geometric axis bending provided:

- a. The yield moment shall be based on use of 0.80 of the geometric axis section modulus.
- b. With maximum compression of the angle-leg tips, the nominal flexural strength M_n shall be determined by the provisions in Section 5.1.1 and in Section 5.1.3,

where

$$M_{ob} = \frac{0.66Eb^4tC_b}{I^2} \left[\sqrt{1 + 0.78(lt/b^2)^2} - 1 \right] \quad (5-4)$$

l = unbraced length

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \leq 1.5$$

where

M_{\max} = absolute value of maximum moment in the unbraced beam segment

M_A = absolute value of moment at quarter point of the unbraced beam segment

M_B = absolute value of moment at centerline of the unbraced beam segment

M_C = absolute value of moment at three-quarter point of the unbraced beam segment

- c. With maximum tension at the angle-leg tips, the nominal flexural strength shall be determined according to Section 5.1.2 and in Section 5.1.3 using M_{ob} in Equation 5-4 with -1 being replaced by $+1$.

5.2.3. Unequal-leg angle members without lateral-torsional restraint subjected to bending about one of the geometric axes shall be designed using Section 5.3.

5.3. Bending about Principal Axes

Angles without lateral-torsional restraint shall be designed considering principal-axis bending, except for the alternative of Section 5.2.2, if appropriate. Bending about both of the principal axes shall be evaluated as required in Section 6.

5.3.1. Equal-leg angles:

- a. Major-axis bending:

The nominal flexural strength M_n about the major principal axis shall be determined by the provisions in Section 5.1.1 and in Section 5.1.3,

where

$$M_{ob} = C_b \frac{0.46Eb^2t^2}{l} \quad (5-5)$$

- b. Minor-axis bending:

The nominal design strength M_n about the minor principal axis shall be determined by Section 5.1.1 when the leg tips are in compression, and by Section 5.1.2 when the leg tips are in tension.

5.3.2. Unequal-leg angles:

- a. Major-axis bending:

The nominal flexural strength M_n about the major principal axis shall be determined by the provisions in Section 5.1.1 for the compression leg and in Section 5.1.3,

where

$$M_{ob} = 4.9E \frac{I_z}{l^2} C_b \left[\sqrt{\beta_w^2 + 0.052(l/r_z)^2} + \beta_w \right] \quad (5-6)$$

I_z = minor principal axis moment of inertia

r_z = radius of gyration for minor principal axis

$\beta_w = \left[\frac{1}{I_w} \int_A z_o(w^2 + z^2) dA \right] - 2z_o$, special section property for unequal-leg angles, positive for short leg in compression and negative for long leg in compression (see Commentary for values for common angle sizes). If the long leg is in compression anywhere along the unbraced length of the member, the negative value of β_w shall be used.

z_o = coordinate along z axis of the shear center with respect to centroid

I_w = moment of inertia for major principal axis

b. Minor-axis bending:

The nominal design strength M_n about the minor principal axis shall be determined by Section 5.1.1 when leg tips are in compression and by Section 5.1.2 when the leg tips are in tension.

6. COMBINED FORCES

The interaction equation shall be evaluated for the principal bending axes either by addition of all the maximum axial and flexural terms, or by considering the sense of the associated flexural stresses at the critical points of the cross section, the flexural terms are either added to or subtracted from the axial load term.

6.1. Members in Flexure and Axial Compression

6.1.1. The interaction of flexure and axial compression applicable to specific locations on the cross section shall be limited by Equations 6-1a and 6-1b:

$$\text{For } \frac{P_u}{\phi P_n} \geq 0.2$$

$$\left| \frac{P_u}{\phi P_n} + \frac{8}{9} \left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}} \right) \right| \leq 1.0 \quad (6-1a)$$

$$\text{For } \frac{P_u}{\phi P_n} \leq 0.2$$

$$\left| \frac{P_u}{2\phi P_n} + \left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}} \right) \right| \leq 1.0 \quad (6-1b)$$

where

P_u = required compressive strength

P_n = nominal compressive strength determined in accordance with Section 4

M_u = required flexural strength

M_n = nominal flexural strength for tension or compression in accordance with Section 5, as appropriate. Use section modulus for specific location in the cross section and consider the type of stress.

ϕ = ϕ_c = resistance factor for compression = 0.90

ϕ_b = resistance factor for flexure = 0.90

w = subscript relating symbol to major-axis bending

z = subscript relating symbol to minor-axis bending

In Equations 6-1a and 6-1b when M_n represents the flexural strength of the compression side, the corresponding M_u shall be multiplied by B_1 .

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \geq 1.0 \quad (6-2)$$

where

C_m = bending coefficient defined in AISC LRFD

P_{e1} = elastic buckling load for the braced frame defined in AISC LRFD

6.1.2. For members constrained to bend about a geometric axis with nominal flexural strength determined per Section 5.2.1, the radius of gyration r for P_{e1} shall be taken as the geometric axis value. The bending terms for the principal axes in Equations 6-1a and 6-1b shall be replaced by a single geometric axis term.

6.1.3. Alternatively, for equal-leg angles without lateral-torsional restraint along the length and with bending applied about one of the geometric axes, the provisions of Section 5.2.2 are permitted for the required and design bending strength. If Section 5.2.2 is used for M_n , the radius of gyration about the axis of bending r for P_{e1} shall be taken as the geometric axis value of r divided by 1.35 in the absence of a more detailed analysis. The bending terms for the principal axes in Equations 6-1a and 6-1b shall be replaced by a single geometric axis term.

6.2. Members in Flexure and Axial Tension

The interaction of flexure and axial tension shall be limited by Equations 6-1a and 6-1b where

P_u = required tensile strength

P_n = nominal tensile strength determined in accordance with Section 2

M_u = required flexural strength

M_n = nominal flexural strength for tension or compression in accordance with Section 5, as appropriate. Use section modulus for specific location in the cross section and consider the type of stress.

ϕ = ϕ_t = resistance factor for tension = 0.90

ϕ_b = resistance factor for flexure = 0.90

For members subject to bending about a geometric axis, the required bending strength evaluation shall be in accordance with Sections 6.1.2 and 6.1.3. Second-order effects due to axial tension and bending interaction are permitted to be considered in the determination of M_u for use in Formulas 6-1a and 6-1b. In lieu of using Formulas 6-1a and 6-1b, a more detailed analysis of the interaction of flexure and tension is permitted.

COMMENTARY

on the Load and Resistance Factor Design Specification for Single-Angle Members

November 10, 2000

INTRODUCTION

This Specification is intended to be complete for normal design usage in conjunction with the main 1999 AISC LRFD Specification and Commentary.

This Commentary furnishes background information and references for the benefit of the engineer seeking further understanding of the derivation and limits of the specification.

The Specification and Commentary are intended for use by design professionals with demonstrated engineering competence.

C2. TENSION

The criteria for the design of tension members in AISC LRFD Specification Section D1 have been adopted for angles with bolted connections. However, recognizing the effect of shear lag when the connection is welded, the criteria in Section B3 of the AISC LRFD Specification have been applied.

The advisory upper slenderness limits are not due to strength considerations but are based on professional judgment and practical considerations of economics, ease of handling, and transportability. The radius of gyration about the z axis will produce the maximum l/r and, except for very unusual support conditions, the maximum Kl/r . Since the advisory slenderness limit for compression members is less than for tension members, an accommodation has been made for members with $Kl/r > 200$ that are always in tension, except for unusual load conditions which produce a small compression force.

C3. SHEAR

Shear stress due to factored loads in a single-angle member are the result of the gradient in the bending moment along the length (flexural shear) and the torsional moment.

The maximum elastic stress due to flexural shear may be computed by

$$f_v = \frac{1.5V_b}{bt} \quad (\text{C3-1})$$

where

V_b = component of the shear force parallel to the angle leg with length b and thickness t , kips

The stress, which is constant through the thickness, should be determined for both legs to determine the maximum.

The 1.5 factor is the calculated elastic value for equal-leg angles loaded along one of the principal axes. For equal-leg angles loaded along one of the geometric axes (laterally braced or unbraced) the factor is 1.35. Constants between these limits may be calculated conservatively from $V_b Q/I_t$ to determine the maximum stress at the neutral axis.

Alternatively, if only flexural shear is considered, a uniform flexural shear stress in the leg of V_b/bt may be used due to inelastic material behavior and stress redistribution.

If the angle is not laterally braced against twist, a torsional moment is produced equal to the applied transverse load times the perpendicular distance e to the shear center, which is at the heel of the angle cross section. Torsional moments are resisted by two types of shear behavior: pure torsion (St. Venant) and warping torsion (Seaburg and Carter, 1997). If the boundary conditions are such that the cross section is free to warp, the applied torsional moment M_T is resisted by pure shear stresses as shown in Figure C3.1a. Except near the ends of the legs, these stresses are constant along the length of the leg, and the maximum value can be approximated by

$$f_v = M_T t / J = \frac{3M_T}{At} \quad (\text{C3-2})$$

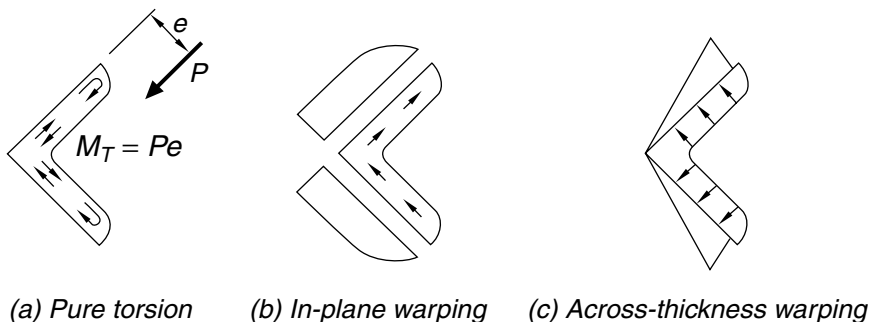


Fig. C.3.1. Shear stresses due to torsion.

where

J = torsional constant (approximated by $\Sigma bt^3/3$ when precomputed value is unavailable)

A = angle cross-sectional area

At a section where warping is restrained, the torsional moment is resisted by warping shear stresses of two types (Gjelsvik, 1981). One type is in-plane (contour) as shown in Figure C3.1b, which varies from zero at the toe to a maximum at the heel of the angle. The other type is across the thickness and is sometimes referred to as secondary warping shear. As indicated in Figure C3.1c, it varies from zero at the heel to a maximum at the toe.

In an angle with typical boundary conditions and an unrestrained load point, the torsional moment produces all three types of shear stresses (pure, in-plane warping, and secondary warping) in varying proportions along its length. The total applied moment is resisted by a combination of three types of internal moments that differ in relative proportions according to the distance from the boundary condition. Using typical angle dimensions, it can be shown that the two warping shears are approximately the same order of magnitude and are less than 20 percent of the pure shear stress for the same torsional moment. Therefore, it is conservative to compute the torsional shear stress using the pure shear equation and total applied torsional moment M_T as if no warping restraint were present. This stress is added directly to the flexural shear stress to produce a maximum surface shear stress near the mid-length of a leg. Since this sum is a local maximum that does not extend through the thickness, applying the limit of $\phi_v 0.6F_y$ adds another degree of conservatism relative to the design of other structural shapes.

In general, torsional moments from laterally unrestrained transverse loads also produce warping normal stresses that are superimposed on bending stresses. However, since the warping strength for a single angle is relatively small, this additional bending effect is negligible and often ignored in design practice.

C4. COMPRESSION

The provisions for the critical compression stress account for the three possible limit states that may occur in an angle column depending on its proportions: general column flexural buckling, local buckling of thin legs, and flexural-torsional buckling of the member. The Q -factor in the equation for critical stress accounts for the local buckling, and the expressions for Q are nondimensionalized from AISC LRFD Specification (AISC, 1999) Appendix B5. Flexural-torsional buckling is covered in Appendix E of the AISC LRFD Specification (AISC, 1999). This strength limit state is approximated by the Q -factor reduction for slender-angle legs. For non-slender sections where $Q = 1$, flexural-torsional buckling is relevant for relatively short columns, but it was shown by Galambos (1991) that the error of neglecting this effect is not significant. For this reason no explicit consideration of this effect is required in

these single-angle specifications. The provisions of Appendix E of AISC LRFD may be conservatively used to directly consider flexural-torsional buckling for single-angle members.

The effective length factors for angle columns may be determined by consulting the paper by Lutz (1992).

The resistance factor ϕ was increased from 0.85 in AISC LRFD for all cross sections to 0.90 for single angles only because it was shown that a ϕ of 0.90 provides an equivalent degree of reliability (Galambos, 1992).

C5. FLEXURE

Flexural strength limits are established for yielding, local buckling, and lateral-torsional buckling. In addition to addressing the general case of unequal-leg single angles, the equal-leg angle is treated as a special case. Furthermore, bending of equal-leg angles about a geometric axis, an axis parallel to one of the legs, is addressed separately as it is a very common situation.

The tips of an angle refer to the free edges of the two legs. In most cases of unrestrained bending, the flexural stresses at the two tips will have the same sign (tension or compression). For constrained bending about a geometric axis, the tip stresses will differ in sign. Criteria for both tension and compression at the tip should be checked as appropriate, but in most cases it will be evident which controls.

Appropriate serviceability limits for single-angle beams need also to be considered. In particular, for longer members subjected to unrestrained bending, deflections are likely to control rather than lateral-torsional or local buckling strength.

C5.1.1. These provisions follow the LRFD format for nominal flexural resistance. There is a region of full yielding, a linear transition to the yield moment, and a region of local buckling. The strength at full yielding is limited to a shape factor of 1.50 applied to the yield moment. This leads to a lower bound plastic moment for an angle that could be bent about any axis, inasmuch as these provisions are applicable to all flexural conditions. The 1.25 factor originally used was known to be a conservative value. Recent research work (Earls and Galambos, 1997) has indicated that the 1.50 factor represents a better lower bound value.

The b/t limits have been modified to be more representative of flexural limits rather than using those for single angles under uniform compression. Typically the flexural stresses will vary along the leg length permitting the use of the stress limits given. Even for the geometric axis flexure case which produces uniform compression along

one leg, use of these limits will provide a conservative value when compared to the results obtained by Earls and Galambos, 1997.

C5.1.2. Since the shape factor for angles is in excess of 1.50, the nominal design strength $M_n = 1.5M_y$ for compact members is justified provided that instability does not control.

C5.1.3. Lateral-torsional instability may limit the flexural strength of an unbraced single-angle beam. As illustrated in Figure C5.1, Equation 5-3a represents the elastic buckling portion with the nominal flexural strength, M_n , varying from 75 percent to 92 percent of the theoretical strength, M_{ob} . Equation 5-3b represents the inelastic buckling transition expression between $0.75M_y$ and $1.5M_y$. Equation 5-3b has been modified to better reflect its use with the increased upper limit of $1.5M_y$. The maximum beam flexural strength $M_n = 1.5M_y$ will occur when the theoretical buckling moment M_{ob} reaches or exceeds $7.7M_y$, as illustrated in Figure C5.1. These equations are modifications of those developed from the results of Australian research on single angles in flexure and on an analytical model consisting of two rectangular elements of length equal to the actual angle leg width minus one-half the thickness (Leigh and Lay, 1984; Australian Institute of Steel Construction, 1975; Leigh and Lay, 1978; Madugula and Kennedy, 1985).

A more general C_b moment gradient formula consistent with the 1999 AISC LRFD Specification is used to correct lateral-torsional stability equations from the assumed most severe case of uniform moment throughout the unbraced length ($C_b = 1.0$). The equation for

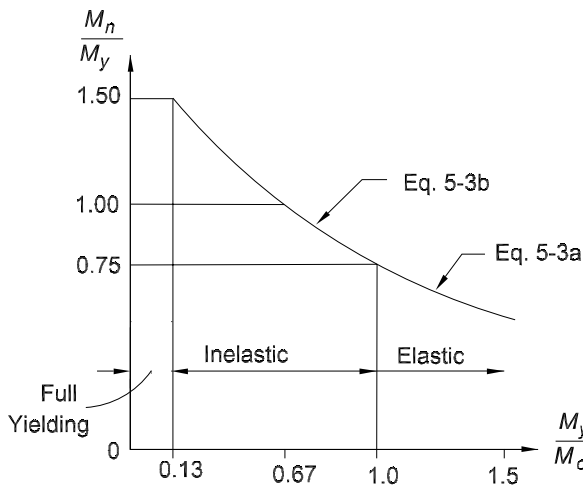


Fig. C5.1. Lateral-torsional buckling limits of a single-angle beam.

C_b used in the ASD version is applicable only to moment diagrams that are straight lines between brace points. In lieu of a more detailed analysis, the reduced maximum limit of 1.5 is imposed for single-angle beams to represent conservatively the lower envelope of this cross section's non-uniform bending response.

- C5.2.1.** An angle beam loaded parallel to one leg will deflect and bend about that leg only if the angle is restrained laterally along the length. In this case simple bending occurs without any torsional rotation or lateral deflection and the geometric axis section properties should be used in the evaluation of the flexural design strength and deflection. If only the point of maximum moment is laterally braced, lateral-torsional buckling of the unbraced length under simple bending must also be checked, as outlined in Section 5.2.1b.
- C5.2.2.** When bending is applied about one leg of a laterally unrestrained single angle, it will deflect laterally as well as in the bending direction. Its behavior can be evaluated by resolving the load and/or moments into principal axis components and determining the sum of these principal axis flexural effects. Section 5.2.2 is provided to simplify and expedite the design calculations for this common situation with equal-leg angles.

For such unrestrained bending of an equal-leg angle, the resulting maximum normal stress at the angle tip (in the direction of bending) will be approximately 25 percent greater than calculated using the geometric axis section modulus. The value of M_{ob} in Equation 5-4 and the evaluation of M_y using 0.80 of the geometric axis section modulus reflect bending about the inclined axis shown in Figure C5.2.

The deflection calculated using the geometric axis moment of inertia has to be increased 82 percent to approximate the total deflection. Deflection has two components, a vertical component (in the direction of applied load) 1.56 times the calculated value and a horizontal component of 0.94 of the calculated value. The resultant total deflection is in the general direction of the weak principal axis bending of the angle (see Figure C5.2). These unrestrained bending deflections should be considered in evaluating serviceability and will often control the design over lateral-torsional buckling.

The horizontal component of deflection being approximately 60 percent of the vertical deflection means that the lateral restraining force required to achieve purely vertical deflection (Section 5.2.1) must be 60 percent of the applied load value (or produce a moment 60 percent of the applied value) which is very significant.

Lateral-torsional buckling is limited by M_{ob} (Leigh and Lay, 1984 and 1978) in Equation 5-4, which is based on

$$M_{cr} = \frac{2.33Eb^4t}{(1 + 3\cos^2\theta)(KI)^2} \times \left[\sqrt{\sin^2\theta + \frac{0.156(1 + 3\cos^2\theta)(KI)^2t^2}{b^4}} + \sin\theta \right] \quad (C5-1)$$

(the general expression for the critical moment of an equal-leg angle) with $\theta = -45^\circ$ for the condition where the angle tip stress is compression (see Figure C5.3). Lateral-torsional buckling can also limit the moment capacity of the cross section when the maximum angle tip stress is tension from geometric axis flexure, especially with use of the new flexural capacity limits in Section 5.1. Using $\theta = 45^\circ$ in Equation C5-1, the resulting expression is Equation 5-4 with a +1 instead of -1 as the last term.

Stress at the tip of the angle leg parallel to the applied bending axis is of the same sign as the maximum stress at the tip of the other leg when the single angle is unrestrained. For an equal-leg angle this stress is about one-third of the maximum stress. It is only necessary to check the nominal bending strength based on the tip of the angle leg with the maximum stress when evaluating such an angle. Since this maximum moment per Section 5.2.2 represents combined principal axis moments and Equation 5-4 represents the design limit for these combined flexural moments, only a single flexural term needs to be considered when evaluating combined flexural and axial effects.

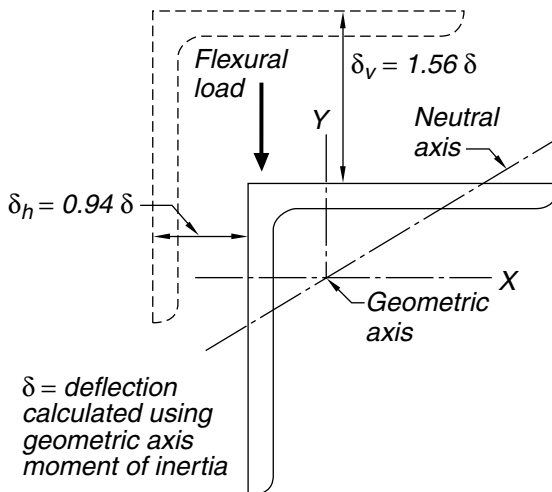


Fig. C5.2. Geometric axis bending of laterally unrestrained equal-leg angles.

C5.2.3. For unequal-leg angles without lateral-torsional restraint the applied load or moment must be resolved into components along the two principal axes in all cases and designed for biaxial bending using the interaction equation.

C5.3.1. Under major axis bending of equal-leg angles Equation 5-5 in combination with 5-3a or 5-3b controls the nominal design moment against overall lateral-torsional buckling of the angle. This is based on M_{cr} , given earlier with $\theta = 0$.

Lateral-torsional buckling for this case will reduce the stress below $1.5M_y$ only for $l/t \geq 7350C_b/F_y$ ($M_{ob} = 7.7M_y$). If the l/t^2 parameter is small (less than approximately $0.87C_b$ for this case), local buckling will control the nominal design moment and M_n based on lateral-torsional buckling need not be evaluated. Local buckling must be checked using Section 5.1.1.

C5.3.2. Lateral-torsional buckling about the major principal W axis of an unequal-leg angle is controlled by M_{ob} in Equation 5-6. Section property β_w reflects the location of the shear center relative to the principal axis of the section and the bending direction under uniform bending. Positive β_w and maximum M_{ob} occurs when the shear center is in flexural compression while negative β_w and minimum M_{ob} occurs when the shear center is in flexural tension (see Figure C5.4). This β_w effect is consistent with behavior of singly symmetric I-shaped beams which are more stable when the compression flange is larger than the tension flange. For principal W -axis bending of equal-leg angles, β_w is equal to zero due to symmetry and Equation 5-6 reduces to Equation 5-5 for this special case.

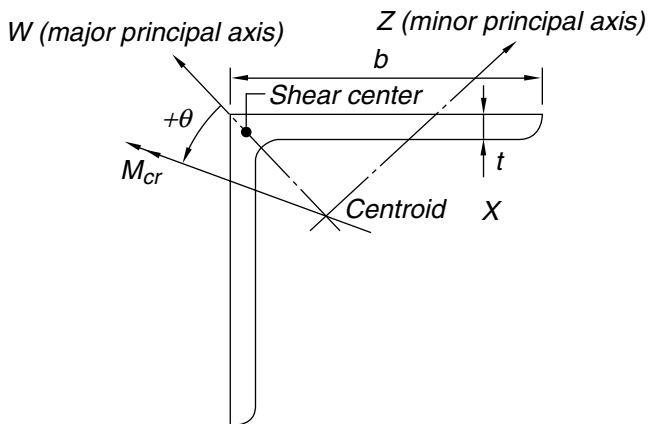


Fig. C5.3. Equal-leg angle with general moment loading.

TABLE C5.1
 β_w Values for Angles

Angle Size (in.)	β_w (in.)*
9 × 4	6.54
8 × 6	3.31
8 × 4	5.48
7 × 4	4.37
6 × 4	3.14
6 × 3.5	3.69
5 × 3.5	2.40
5 × 3	2.99
4 × 3.5	0.87
4 × 3	1.65
3.5 × 3	0.87
3.5 × 2.5	1.62
3 × 2.5	0.86
3 × 2	1.56
2.5 × 2	0.85
Equal legs	0.00

* Has positive or negative value depending on direction of bending (see Figure C5.4).

For reverse curvature bending, part of the unbraced length has positive β_w , while the remainder has negative β_w , and conservatively, the negative value is assigned for that entire unbraced segment.

β_w is essentially independent of angle thickness (less than one percent variation from mean value) and is primarily a function of the leg widths. The average values shown in Table C5.1 may be used for design.

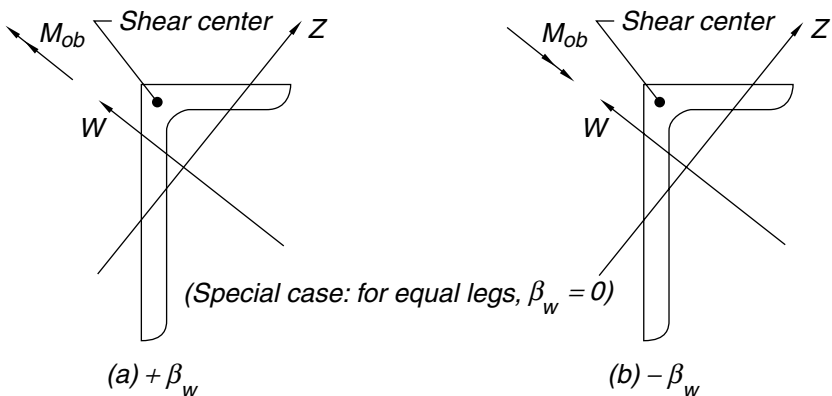


Fig. C5.4. Unequal-leg angle in bending.

C6. COMBINED STRESSES

The stability and strength interaction equations of AISC LRFD Specification Chapter H have been adopted with modifications to account for various conditions of bending that may be encountered. Bending will usually accompany axial loading in a single-angle member since the axial load and connection along the legs are eccentric to the centroid of the cross section. Unless the situation conforms to Section 5.2.1 or 5.2.2 in that Section 6.1.2 or 6.1.3 may be used, the applied moment should be resolved about the principal axes for the interaction check.

For the non-symmetric and singly symmetric single angles, the interaction expression related to stresses at a particular location on the cross section is the most accurate due to lack of double symmetry. At a particular location, it is possible to have stresses of different sign from the various components such that a combination of tensile and compressive stress will represent a critical condition. The absolute value of the combined terms must be checked at the angle-leg tips and heel and compared with 1.0.

When using the combined force expressions for single angles, M_{uw} and M_{uz} are positive as customary. The evaluation of M_n in Section 5.1 is dependent on the location on the cross section being examined by using the appropriate value of section modulus, S . Since the sign of the stress is important in using Equations 6-1a and 6-1b, M_n is considered either positive or negative by assigning a sign to S to reflect the stress condition as adding to, or subtracting from, the axial load effect. A designer may choose to use any consistent sign convention.

It is conservative to ignore this refinement and simply use positive critical M_n values in the bending terms and add the absolute values of all terms (Elgaaly, Davids, and Dagher, 1992 and Adluri and Madugula, 1992).

Alternative special interaction equations for single angles have been published (Adluri and Madugula, 1992).

C6.1.3. When the total maximum flexural stress is evaluated for a laterally unrestrained length of angle per Section 5.2, the bending axis is the inclined axis shown in Figure C5.2. The radius of gyration modification for the moment amplification about this axis is equal to $\sqrt{1.82} = 1.35$ to account for the increased unrestrained bending deflection relative to that about the geometric axis for the laterally unrestrained length. The 1.35 factor is retained for angles braced only at the point of maximum moment to maintain a conservative calculation for this case. If the brace exhibits any flexibility permitting lateral movement of the angle, use of $r = r_x$ would not be conservative.

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